

INCLUSION, DISJOINTNESS AND CHOICE: THE LOGIC OF LINGUISTIC CLASSIFICATION

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Abstract

We investigate the logical structure of concepts generated by conjunction and disjunction over a monotonic multiple inheritance network where concept nodes represent linguistic categories and links indicate basic inclusion (ISA) and disjointness (ISNOTA) relations. We model the distinction between primitive and defined concepts as well as between closed- and open-world reasoning. We apply our logical analysis to the sort inheritance and unification system of HPSG and also to classification in systemic choice systems.

Introduction

Our focus in this paper is a stripped-down monotonic inheritance-based knowledge representation system which can be applied directly to provide a clean declarative semantics for Halliday's systemic choice systems (see Winograd 1983, Mellish 1988, Kress 1976) and the inheritance module of head-driven phrase-structure grammar (HPSG) (Pollard and Sag 1987, Pollard in press). Our inheritance networks are constructed from only the most rudimentary primitives: basic concepts and ISA and ISNOTA links. By applying general algebraic techniques, we show how to generate a meet semilattice whose nodes correspond to consistent conjunctions of basic concepts and where meet corresponds to conjunction. We also show how to embed this result in a distributive lattice where the elements correspond to arbitrary conjunctions and disjunctions of basic concepts and where meet and join correspond to conjunction and disjunction, respectively. While we do not consider either role- or attribute-based reasoning

in this paper, our constructions are directly applicable as a front-end for the combined attribute- and concept-based formalisms of Ait-Kaci (1986), Nebel and Smolka (1989), Carpenter (1990), Carpenter, Pollard and Franz (1991) and Pollard (in press).

The fact that terms in distributive lattices have disjunctive normal forms allows us to factor our construction into two stages: we begin with the consistent conjunctive concepts generated from our primitive concepts and then form arbitrary disjunctions of these conjunctions. The conjunctive construction is useful on its own as its result is a semilattice where meet corresponds to conjunction. In particular, the conjunctive semilattice is ideally suited to conjunctive logics such as those employed for unification, as in HPSG.

We will consider the distinction between primitive and defined concepts, a well-known distinction expressible in terminological reasoning systems such as KL-ONE (Brachman 1979, Brachman and Schmolze 1985), and its descendants (such as LOOM (MacGregor 1988) or CLASSIC (Borgida *et al.* 1989)). We also tackle the variety of closed-world reasoning that is necessary for modeling constraint-based grammars such as HPSG. A similar form of closed-world reasoning is supported by LOOM with the *disjoint-covering* construction.

One of the benefits of our notion of inheritance is that it allows us to express the natural semantics of both systemic choice systems and HPSG inheritance hierarchies using basic concepts and ISA and ISNOTA links. In particular, we will see how choice systems correspond to ISNOTA reasoning, multiple choices can be captured in our conjunctive construction and how dependent choices can

be represented by inheritance. One result of our construction will be a demonstration that the systemic classification and HPSG systems are variant graphical representations of the same kind of underlying information regarding inclusion, disjointness and choice.

Inheritance Networks

Our inheritance networks are particularly simple, being constructed from basic concepts and two kinds of “inheritance” links.

Definition 1 (Inheritance Network) *An inheritance net is a tuple $(\text{BasConc}, \text{ISA}, \text{ISNOTA})$ where:*

- **BasConc:** *a finite set of basic concepts*
- **ISA** $\subseteq \text{BasConc} \times \text{BasConc}$: *the basic inclusion relation*
- **ISNOTA** $\subseteq \text{BasConc} \times \text{BasConc}$: *the basic disjointness relation*

The interpretation of a net is straightforward: each basic concept is thought of as representing a set of empirical objects, where $P \text{ ISA } Q$ means that all P 's are Q 's and $P \text{ ISNOTA } Q$ means that no P 's are Q 's. Our primary interest is in the logical relationships between concepts rather than in the actual extensions of the concepts themselves. This is in accord with standard linguistic practice, where the focus is on types of utterances rather than utterance tokens. An example of an inheritance network is given in Figure 1. We have followed the standard convention of placing the more specific elements toward the bottom of the network, with arrows indicating the directionality of the ISA links (for instance, $d \text{ ISA } f$ and $b \text{ ISNOTA } c$).

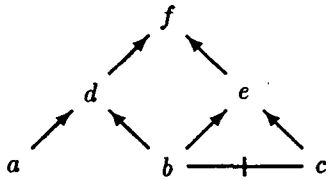


Figure 1: Inheritance Hierarchy

We can automatically deduce all of the inclusion and disjointness relations that follow from the basic ones (Carpenter and Thomason 1990).

Definition 2 (Inclusion/Disjointness) *The inclusion relation $\text{ISA}^* \subseteq \text{BasConc} \times \text{BasConc}$ is the smallest such that:*

- $P \text{ ISA}^* P$ (Reflexive)
- if $P \text{ ISA } Q$ and $Q \text{ ISA}^* R$ then $P \text{ ISA}^* R$ (Transitive)

The disjointness relation $\text{ISNOTA}^ \subseteq \text{BasConc} \times \text{BasConc}$ is the smallest such that:*

- if $P \text{ ISNOTA } Q$ or $Q \text{ ISNOTA } P$ then $P \text{ ISNOTA}^* Q$ (Symmetry)
- if $P \text{ ISA}^* Q$ and $Q \text{ ISNOTA}^* R$ then $P \text{ ISNOTA}^* R$ (Chaining)

These derived inclusion and disjointness relations express all of the information that follows from the basic relations. In particular, ISA^* is the smallest pre-order extending ISA . For convenience, we allow concepts P such that $P \text{ ISNOTA}^* P$; any such inconsistent concepts are automatically filtered out by the conjunctive construction. Similarly, we allow concepts P and Q such that $P \text{ ISA}^* Q$ and $Q \text{ ISA}^* P$. In this case, P and Q are merged during the conjunctive construction so that they behave identically.

Conjunctions

A conjunctive concept is modeled as a set $\mathbf{P} \subseteq \text{BasConc}$ of basic concepts. A conjunctive concept \mathbf{P} corresponds to the conjunction of the concepts $P \in \mathbf{P}$; an object is a \mathbf{P} if and only if it is a P for every $P \in \mathbf{P}$. But arbitrary sets of basic concepts are not good models for conjunctive concepts; we need to identify conjunctive concepts which convey identical information and also remove those conjunctive concepts which are inconsistent. We address the first issue by requiring conjunctive concepts to be closed under inheritance and the second by removing any concepts which contain a pair of disjoint basic concepts.

Definition 3 (Conjunctive Concept) *A set $\mathbf{P} \subseteq \text{BasConc}$ is a conjunctive concept if:*

- if $P \in \mathbf{P}$ and $P \text{ ISA}^* P'$ then $P' \in \mathbf{P}$
- no $P, P' \in \mathbf{P}$ are such that $P \text{ ISNOTA}^* P'$

Let ConjConc be the set of conjunctive concepts.

There is a natural inclusion or specificity ordering on our conjunctive concepts; if $P \subseteq Q$ then every object which can be classified as a Q can also be classified as a P . The conjunctive concepts derived from the inheritance net in Figure 1 are displayed in Figure 2, where we have $P \subseteq Q$ for every derived “ISA” arc $Q \rightarrow P$.

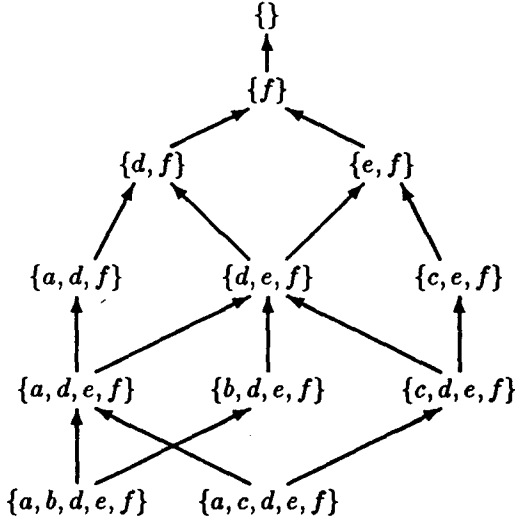


Figure 2: Conjunctive Concept Ordering

Defined Concepts

So far, we have considered only *primitive* basic concepts. A *defined* basic concept is taken to be fully determined by its set of superconcepts (in the general terminological case with roles, restrictions on role values can also contribute to the definition of a concept (Brachman and Schmolze 1985)). In particular, a defined basic concept P is assumed to carry the same information as the conjunction of all of the concepts P' such that $P \text{ ISA } P'$. For example, consider the basic concept b in Figure 1. The conjunctive concept $\{b, d, e, f\}$ is strictly more informative than $\{d, e, f\}$; the primitiveness of b allows for the possibility that there is information to be gained from knowing that an object is a b that can not be gained from knowing that it is both a d and an e . On the other hand, if we assume that b is defined, then the presence of d and e in a conjunctive concept should ensure the presence of b , thus eliminating the sets $\{d, e, f\}$, $\{c, d, e, f\}$ and $\{a, d, e, f\}$ from consideration, as they are equivalent to the conjunctive concepts

$\{b, d, e, f\}$, $\{b, c, d, e, f\}$ and $\{a, b, d, e, f\}$ respectively. In the primitive case, being a d and an e is a *necessary* condition for being a b ; in the defined case, being a d and e is also a *sufficient* condition for being a b .

In general, suppose that $\text{DefConc} \subseteq \text{BasConc}$ is the subset of *defined concepts*. To account for this new information, we add the following additional clause to the conditions that P must satisfy to be a conjunctive concept:

- (1) If $P \in \text{DefConc}$ and $\{P' \mid P \neq P' \text{ and } P \text{ ISA } P'\} \subseteq P$ then $P \in \mathcal{P}$.

With the example in Figure 1 and the assumption that $\text{DefConc} = \{b, f\}$, we generate the conjunctive concepts in Figure 3. We have adopted the condition of only displaying the maximally specific primitive concepts of a conjunctive concept, as the other basic concepts can be determined from these. Note that the assumption that f , the most

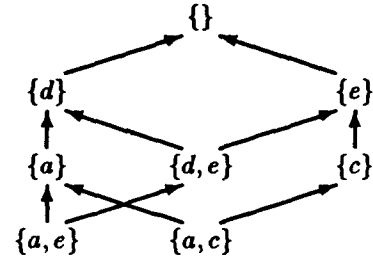


Figure 3: Conjunctive Construction with Defined Concepts

general basic concept, is defined means that every conjunctive concept must contain f , because the set $\{P \mid f \neq P \text{ and } f \text{ ISA } P\}$ is empty and thus a subset of every conjunctive concept. Thus $\{\}$ is equivalent to $\{f\}$ in terms of conjunctive information so that every object is classified as an f .

The set of conjunctive concepts ordered by reverse set inclusion has the pleasant property of being closed under consistent meets, where the meet operation represents conjunction (“unification”). More precisely, a set $\mathcal{P} \subseteq \text{ConjConc}$ of conjunctive concepts is *consistent* if there is a conjunctive concept P which contains all of the concepts contained in the conjunctive concepts in \mathcal{P} so that $\bigcup \mathcal{P} \subseteq P$. The following theorem states that for every consistent set \mathcal{P} of concepts, there is a least P such that $P \supseteq \bigcup \mathcal{P}$. This least P is written $\bigsqcup \mathcal{P}$

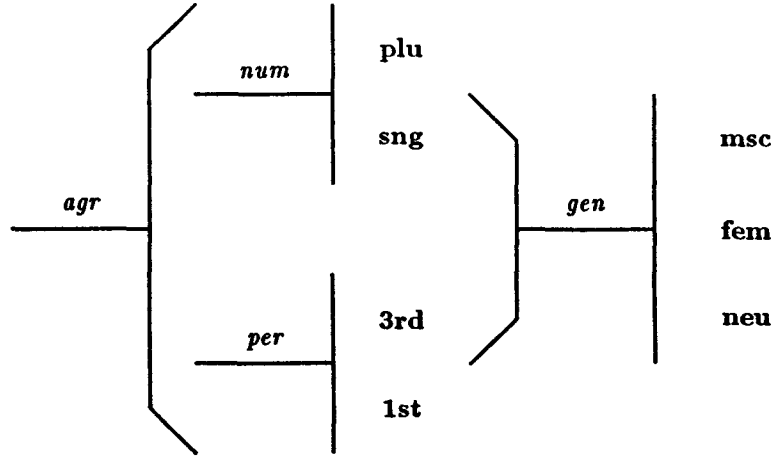


Figure 4: Systemic Choice Network

and called the *meet* of \mathcal{P} .

Theorem 4 *The meet in $\langle \text{ConjConc}, \supseteq \rangle$ for a consistent set $\mathcal{P} \subseteq \text{ConjConc}$ of conjunctive concepts is given by:*

$$\begin{aligned}
 & \sqcap \mathcal{P} \\
 = & \bigcap \{ P' \in \text{ConjConc} \mid P' \supseteq P \text{ for each } P \in \mathcal{P} \} \\
 = & \bigcap \{ P' \in \text{ConjConc} \mid P' \supseteq \bigcup \mathcal{P} \} \\
 = & \{ P \in \text{BasConc} \mid \text{for every } P' \in \text{ConjConc}, \\
 & \quad P' \supseteq \bigcup \mathcal{P} \text{ implies } P \in P' \}
 \end{aligned}$$

Proof: This is an immediate consequence of the fact that ConjConc is closed under arbitrary intersections. \square

Another way to generate the meet of a collection of conjunctive concepts is to close their union under inheritance and concept definition. It should be observed that joins (intersections), while always existing, in general represent only informational generalizations, not necessarily disjunctions.

Systemic Choice Systems

Mellish (1988) showed how the concepts expressible using a systemic choice network such as that found in Figure 4 can be embedded into the lattice of first-order terms with conjunction represented by unification. Our characterization of the concepts expressible in a systemic net instead relies on the translation of systemic notation into an inheritance network with ISA and ISNOTA links.

The resulting conjunctive concepts correspond to the concepts that can be expressed in the systemic net. An example of a systemic choice network in the notation of Mellish (1988), is Figure 4. The connective $|$, of which there are three in the diagram, signals disjoint alternatives; for instance, the connective for gender is taken to indicate that a gender must be exactly one of masculine, feminine or neuter. The connective $\{$, of which there is one before gender, indicates necessary preconditions for a choice; in this case, a gender is only chosen if the number is singular and the person is third. Finally, the connective $\{$, of which there is one labeled *agr*, indicates that a choice for an agreement value requires a choice for both number and person.

We construct an inheritance hierarchy from a systemic network by taking a basic primitive concept for every choice in the network. The *choices* in Figure 4 are those items in bold face; the italicized items simply label connectives and are only for convenience (alternatively, we could take the italicized elements to be defined basic concepts). The ISNOTA relation between basic concepts is defined so that P ISNOTA Q if P and Q are connected by the choice connective $|$. For example, we have **3rd** ISNOTA **1st** and **msc** ISNOTA **neu**. Finally, the ISA relation is defined so that if P is one of the choices for a connective which has a precondition P' attached to it, then we include P ISA P' . For instance, we have **msc** ISA *sng* and **msc** ISA *3rd*.

In Figure 5, we display the conjunctive concepts

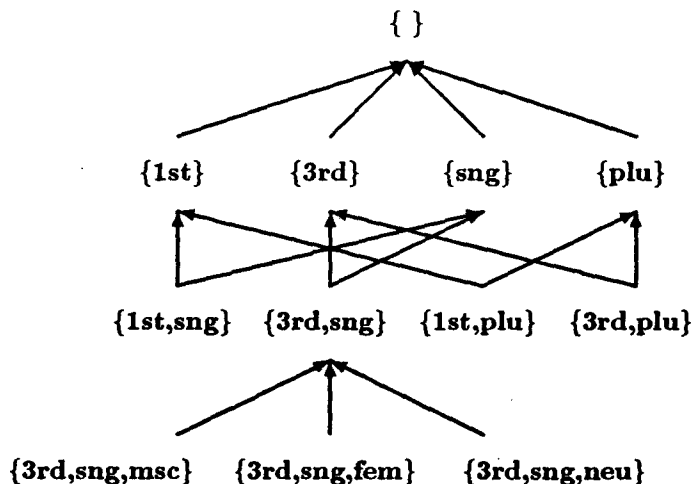


Figure 5: Systemic Choices

generated by the inheritance net stemming from the choice system in Figure 4. A fully determined choice in a choice system corresponds to a maximally specific conjunctive concept, of which there are six in Figure 5.

Sort Inheritance in HPSG

An example of an HPSG sort inheritance hierarchy which represents the same information as the systemic choice system in Figure 4, in the notation of Pollard and Sag (1987), is given in Figure 6. The basic principle behind the HPSG notation is that the bold elements correspond to basic concepts, while the boxed elements correspond to *partitions*, so-called because the concepts in a partition are both pairwise disjoint and exhaustive. In terms of an inheritance network, the elements of a partition (those concepts directly below the partition in the diagram) are related by basic ISNOTA links. For instance, we would have **plu** ISNOTA **sng**. Each partition may also have dependencies which must be fulfilled for the choice to be made; in our case, before an element of the gender partition is chosen, singular must be chosen for number and third for person. These dependencies generate our basic ISA relation. For instance, we must have **plu** ISA **agr** and **fem** ISA **sng**. Carrying out this translation of the HPSG notation into an inheritance net produces to the same result as the translation of the systemic choice system in Figure 4, thus generating the conjunctive concept hierarchy in Figure 5.

In HPSG, it is useful to allow sorts to be defined by conjunction. An example is **main** \wedge **base** \wedge **strict-trans**, which classifies the inputs to the passivization lexical rule (Pollard and Sag 1987:211). Translating the example to our system produces a defined conjunctive concept corresponding to the conjunction of those three basic concepts. On the other hand, a primitive sort such as **aux** cannot be defined as the conjunction of the sorts from which it inherits, namely **verb** and **intrans-raising**, because auxiliaries are not the only intransitive raising verbs. In the hierarchy in Figure 6, it is most natural to consider the basic concept **agr** to be defined rather than primitive; it could simply be eliminated with the same effect. However, in the context of a grammar, **agr** would be one of many possible basic sorts (others being **boolean**, **verb-form**, etc.) and would thus not be eliminable.

Disjunctive Concepts

While meets in the conjunctive concept ordering represent conjunction, joins (intersections) do not represent disjunction. For instance, $\{\mathbf{msc}\} \sqcup \{\mathbf{fem}\} = \{\mathbf{msc}\} \sqcup \{\mathbf{neu}\} = \{\mathbf{3rd, sng}\}$, but the information that an object is masculine or feminine is different than the information that it is masculine or neuter, and more specific than the information that it is simply third-singular. The granularity of the original network dramatically affects the disjunctive concepts which can be rep-

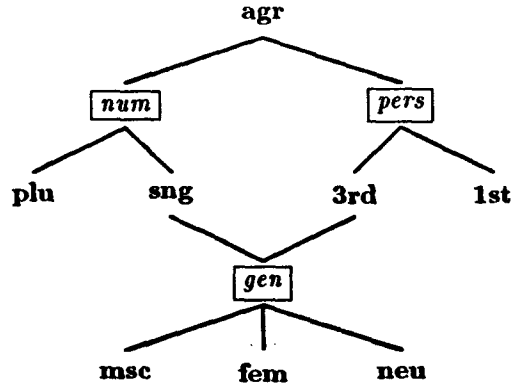


Figure 6: HPSG Inheritance Network Notation

resented (see Borgida and Etherington 1989). For example, we could have partitioned gender into **animate** and **neu** concepts and then partitioned the **animate** concept into **msc** and **fem**. This move would distinguish the join of **msc** and **fem** from the join of **msc** and **neu**.

To complete our study of the logic of simple inheritance, we employ a well-known lattice-theoretic technique for embedding a partial order into a distributive lattice; when applied to conjunctive concept hierarchies, the result is a distributive lattice where concepts correspond to arbitrary conjunctions and disjunctions of basic concepts with joins and meets representing disjunction and conjunction.

We model a disjunctive concept as a set $\mathcal{P} \subseteq \text{ConjConc}$ of conjunctive concepts interpreted disjunctively; an object is classified as a \mathcal{P} just in case it can be classified as a \mathbf{P} for some $\mathbf{P} \in \mathcal{P}$. As with the conjunctive concepts, we identify disjunctive concepts which convey the same information. In this case, we can add more specific concepts to a disjunctive concept \mathcal{P} without affecting its information content.

Definition 5 (Disjunctive Concepts) *A subset $\mathcal{P} \subseteq \text{ConjConc}$ of conjunctive concepts is said to be a disjunctive concept if whenever $\mathbf{P}, \mathbf{Q} \in \text{ConjConc}$ are such that $\mathbf{Q} \supseteq \mathbf{P}$ and $\mathbf{P} \in \mathcal{P}$ then $\mathbf{Q} \in \mathcal{P}$.*

Let DisjConc be the collection of disjunctive concepts.

The inclusion ordering between disjunctive concepts represents specificity, but this time if $\mathcal{P} \subseteq \mathcal{Q}$

then \mathcal{P} is at least as specific as \mathcal{Q} , as \mathcal{Q} admits as many possibilities as \mathcal{P} . Note that the upper-closed sets of a partial ordering form a distributive lattice when ordered by inclusion, since it is a sublattice of a powerset lattice.

Proposition 6 *The structure $(\text{DisjConc}, \subseteq)$ is a distributive lattice.*

Unions (joins) represent disjunctions in DisjConc . Likewise, intersections (meets) represent conjunctions. Furthermore, the function ϕ that maps a conjunctive concept \mathbf{P} to the disjunctive concept $\phi(\mathbf{P}) = \{\mathbf{P}' \mid \mathbf{P}' \supseteq \mathbf{P}\}$ is an embedding of ConjConc into DisjConc that preserves existing meets, so that $\phi(\mathbf{P} \sqcap \mathbf{P}') = \phi(\mathbf{P}) \sqcap \phi(\mathbf{P}')$. Note that this embedding coincides with the standard embedding of a domain into its upper (Smyth) powerdomain (Gunter and Scott in press), with the only difference being that we have reversed the orders of both domains (with the informationally more specific elements toward the bottom), as is conventional in inheritance networks.

More than 30 disjunctive concepts result from the conjunctive concepts in Figure 3, so we will not provide a graphic display of the results of the disjunctive construction applied to a realistic example (for examples of the general construction, see Davey and Priestley 1990).

Closed World Reasoning

In HPSG, Pollard and Sag (1987) partition the concept **sign** into two sub-concepts, **phrase** and

word. This arrangement generates the conjunctive concepts $\{\text{sign}\}$, $\{\text{phrase}\}$ and $\{\text{word}\}$. Applying the disjunctive construction to this result, though, gives us a disjunctive concept $\{\{\text{word}\}, \{\text{phrase}\}\}$ which is strictly more informative than $\{\{\text{sign}\}\}$. This distinction demonstrates the *open-world* nature of our construction; it allows for the possibility of signs which are neither words nor phrases. This form of open-world reasoning is the standard in terminological reasoning systems such as KL-ONE or CLASSIC, though LOOM provides a notion of disjoint-covering which provides the kind of closed-world reasoning we require.

In dealing with linguistic grammars, on the other hand, we clearly wish to exclude any expression from signhood that is neither a phrase nor a word; these choices are meant to be exhaustive in a grammar. The fact that signs can be either words or phrases is explicit; what we need is a way to say that nothing else can be a sign.

In general, we require a set $\text{ClosConc} \subseteq \text{BasConc}$ of *closed concepts* to be specified. When constructing the disjunctive concepts, we identify a closed concept with the disjunction of its immediate subconcepts. In particular, we can replace every occurrence of a closed concept with the disjunction of its immediate subconcepts, so that $\{P\}$ and $\{P' \mid P' \text{ ISA } P\}$ are identified. Closed concepts are treated dually to defined concepts; a defined concept is taken to be the conjunction of its immediate superconcepts, while a closed concept is identified with the disjunction of its immediate subconcepts. The simplest way to achieve this effect is to generate the disjunctive concepts from the subset of conjunctive concepts which contain at least one subconcept of every closed concept which they contain. This leads to the following restriction:

- (2) $P \in \text{DisjConc}$ only if for every $P \in P$ and $P \in P \cap \text{ClosConc}$ there is some $P' \in P$ such that $P' \text{ ISA } P$

Thus if $\text{sign} \in \text{ClosConc}$, we would only consider the conjunctive concepts $\{\text{phrase}\}$ and $\{\text{word}\}$; the concept $\{\text{sign}\}$ contains a closed concept sign , but none of its subconcepts. Consequently, the set $\{\{\text{sign}\}\}$ is no longer a disjunctive concept, while $\{\{\text{phrase}\}, \{\text{word}\}\}$ would be allowed (assuming for this example that phrase and word are not themselves closed).

In grammar development, it will often be the case that all but the maximally specific concepts

are closed. In this case, the disjunctive construction will produce the boolean algebra with maximally specific conjunctive concepts as atoms. Such maximally specific conjunctive concepts were simply taken as primitive by King (1989), who generated a boolean algebra of types corresponding to disjunctions of maximal concepts.

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