Neuro-Symbolic Integration Brings Causal and Reliable Reasoning Proofs [*] [†]

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Abstract

Two lines of approaches are adopted for complex reasoning with LLMs. One line of work prompts LLMs with various reasoning structures, while the structural outputs can be naturally regarded as intermediate reasoning steps. Another line of work adopt LLM-free declarative solvers to do the reasoning task, rendering higher reasoning accuracy but lacking interpretability due to the black-box nature of the solvers. Aiming to resolve the trade-off between answer accuracy and interpretability, we present a simple extension to the latter line of work. Specifically, we showcase that the intermediate search logs generated by Prolog interpreters can be accessed and interpreted into human-readable reasoning proofs. As long as LLMs correctly translate problem descriptions into Prolog representations, the corresponding reasoning proofs are ensured to be causal and reliable. On two logical reasoning and one arithmetic reasoning datasets, our framework obtains significant improvements in terms of both answer accuracy and reasoning proof accuracy. Our code is released at https://github.com/DAMO-NLP-SG/CaRing.

1 Introduction

Large language models (LLMs), like LLaMA-3 (Dubey et al., 2024) and GPT-4 (OpenAI, 2023), are shown to be powerful, but they still struggle with structurally complex reasoning problems, such as logical reasoning (Tafjord et al., 2021) and complex arithmetic reasoning (Cobbe et al., 2021; Ribeiro et al., 2023), as shown in Figure 1. Related work argued that such defects result from the fact that existing LLMs fall short on planning (Valmeekam et al., 2023; Huang et al., 2024; Kambhampati et al., 2024). This is predictable, since one should not expect a next-token-prediction LLM to solve an n-step reasoning problem whose reasoning space grows exponentially regarding n.

Existing LLM-based approaches targeting at this issue can be divided into two-fold. One line of work, such as Self-Consistency (Wang et al., 2023), Tree-of-Thoughts (Yao et al., 2023) and RAP (Hao et al., 2023), adopt various search strategies (e.g., Monte Carlo tree search) to iteratively prompt LLMs, expecting that the correct reasoning path could be achieved using proper process supervision that guides the searching process. Their intermediate search steps are explicit, making the reasoning process interpretable. However, they still rely heavily on LLMs themselves to do planning, thus not fully addressing the aforementioned issue. Besides, LLM inference are performed in each searching step, while solving one problem usually requires many search steps, leading to excessive computational cost. Another line of work, such as Logic-LM (Pan et al., 2023) and SatLM (Ye et al., 2023), translate problem descriptions into declarative symbolic representations (i.e., code) and then adopt off-the-shelf solvers to execute the symbolic representations. Such methods enjoy two key advantages: (1) The declarative symbolic representation is closer to the problem description than the reasoning steps are, so it would be easier for LLMs to do problem translation rather than producing the reasoning steps themselves. (2) The reasoning task is undertaken by an external solver, guaranteeing the correctness of the reasoning process as long as the symbolic representations are correct. However, their search steps are implicitly performed within the solver, making it difficult for humans to inspect whether the reasoning process is correct.

Tracking the above, this work presents a framework that gets rid of LLMs when performing the

[*] The work described in this paper is partially supported by a grant from the Research Grant Council of the Hong Kong Special Administrative Region, China (Project Code: 14200620).

[†] This work was supported by Alibaba Group through the Alibaba Innovative Research (AIR) Program (TA2217728).

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Findings of the Association for Computational Linguistics: NAACL 2025, pages 5747–5759

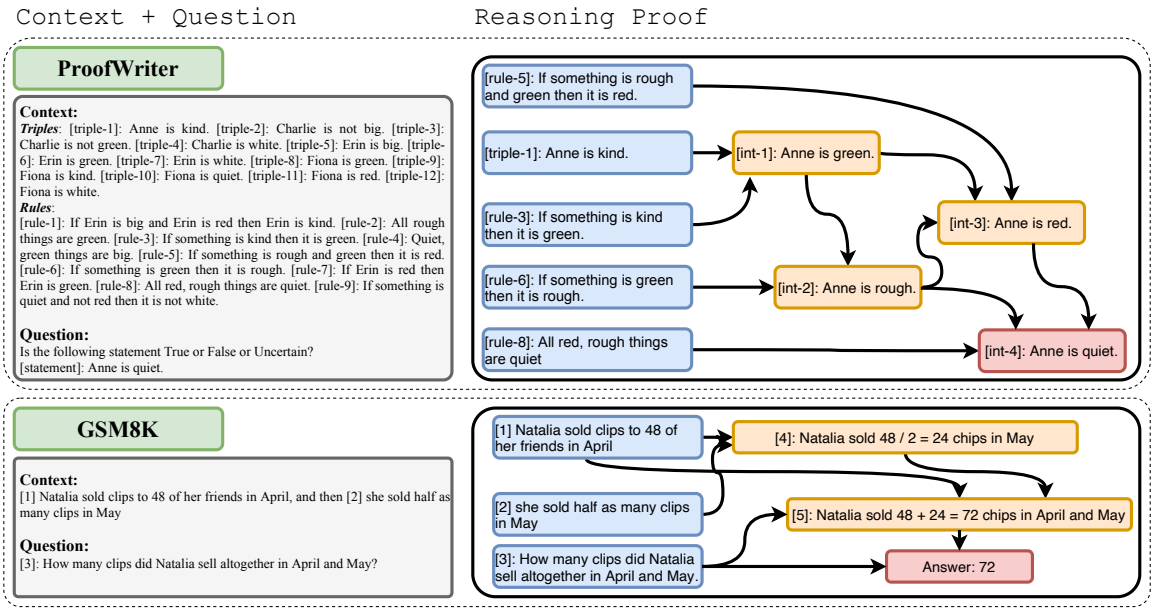


Figure 1: Two examples of complex/structured reasoning problems from ProofWriter and GSM8K, respectively. The reasoning proofs in such problems formulate directed acyclic graphs (DAGs) in a multi-step and multi-premise manner.

reasoning task while still yielding interpretable reasoning steps. Our idea is simple: Searching with symbolic solvers record automatically-generated logs (i.e., trace), which can be translated into human-readable intermediate steps upon further processing. Following this idea, we implement an approach using Prolog, a declarative programming language based on formal logic. We evaluate our approach, CARING (Causal and Reliable Reasoning), on three reasoning datasets that contain reasoning proof annotations, including two logical reasoning datasets, ProofWriter (Tafjord et al., 2021) and PrOntoQA (Saparov and He, 2023), and one arithmetic reasoning dataset, GSM8K (Ribeiro et al., 2023). CARING consistently outperforms the CoT baseline and existing methods in terms of answer accuracy, reasoning proof similarity, and reasoning proof accuracy. On the challenging ProofWriter dataset, CARING using Code-LLaMA-34B yields an answer accuracy of 96.5% and a reasoning proof similarity of 81.0%, while previous SoTA achieved an answer accuracy of 79.7%. Further analysis indicates CARING remains robust when the reasoning problem becomes more complex.

2 Related Work

2.1 Explainable Complex Reasoning

The Chain-of-Thought prompting method, which found out reasoning with LLMs benefits from generating intermediate steps, has sparked a recent trend in how to better do reasoning while remaining explainable. Several works investigated using other reasoning structures, such as trees (Yao et al., 2023; Long, 2023) and graphs (Besta et al., 2023; Zhang et al., 2023). These approaches have shown improved performance, particularly in complex reasoning tasks where the processes involved are often more intricate than simple linear chains. However, despite the alignment of their reasoning proof structures with the gold-standard proofs, these methods still face challenges in ensuring causality and reliability. This limitation stems from their reliance on LLMs for deliberate reasoning, which are prone to hallucinations and may compromise causality.

Some other recent works adopted a less structured manner (Sanyal et al., 2022; Tafjord et al., 2022; Creswell et al., 2023; Kazemi et al., 2023). For example, Selection-Inference (Creswell et al., 2023) divides the reasoning process into two phases: (1) the Selection phase for selecting the premises that might be relevant for the next round of inference, and (2) the Inference phase for conducting a single reasoning step with the selected knowledge fragments.

2.2 Neuro-symbolic Reasoning

Neuro-symbolic systems attempt to leverage the strengths of both neural networks and symbolic reasoning (Andreas et al., 2016; Neelakantan et al.,

2017; Hudson and Manning, 2019; Gupta et al., 2020; Nye et al., 2021). This includes the use of neural networks for pattern recognition and learning from unstructured data, integrated with symbolic systems for rule-based reasoning and knowledge representation. Despite significant progress, neuro-symbolic reasoning faces challenges, notably in scalability and the efficient integration of learning and reasoning components.

Recent advancements in neuro-symbolic research, particularly in reasoning over text, have utilized LLMs to encapsulate knowledge from unstructured human languages, as noted in Lyu et al. (2023); Pan et al. (2023). These methods typically translate natural language into symbolic representations for subsequent execution-based reasoning. However, they have not fully explored the capabilities of symbolic solvers in generating detailed reasoning proofs. In contrast, our approach leverages customized meta-interpreters in conjunction with symbolic solvers to uncover and articulate the underlying reasoning proofs. This not only enhances the transparency of automatic reasoning systems but also simplifies the process for humans to verify their correctness and safety.

3 Method

The problem we are interested in is featured with structured or complex reasoning. As depicted in Figure 1, these problems typically necessitate multi-step and multi-premise reasoning over a directed acyclic graph (DAG), where individual nodes signify distinct knowledge fragments and directed edges denote reasoning steps. Each reasoning step uses existing knowledge to infer new relevant knowledge. Numerous knowledge fragments are often aggregated to infer a new one, which we denote as "multi-premise". The solver usually performs multiple such steps to reach an ultimate goal, which we denote as "multi-step". This entire reasoning process naturally composes a DAG.

We are interested in providing accurate answers along with causal and reliable explanations for such reasoning problems. Kowalski (1979) proposed that *Algorithm* = *Logic* + *Control*, where *Logic* refers to the knowledge which can be used to solve the problem and *Control* refers to the problem-solving strategy in which the knowledge can be used. They further proved that an algorithm benefits from separating the *Logic* component and the *Control* component. Existing works that adopt a symbolic solver to search for valid reasoning paths have followed this design principle (Pan et al., 2023; Ye et al., 2023). Our work also follows this principle.

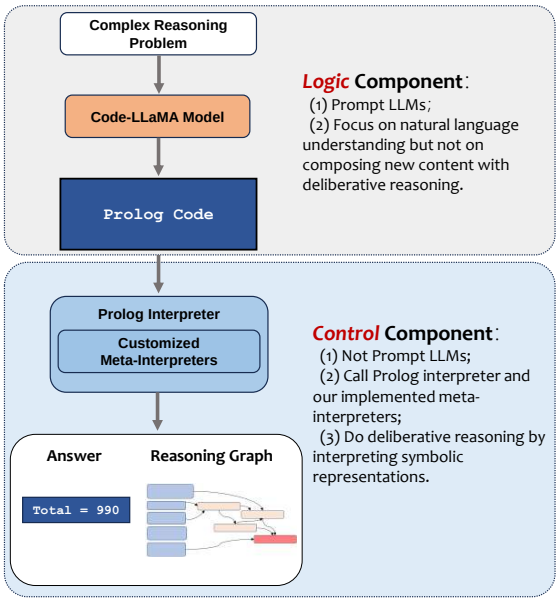


Figure 2: Illustration of our CARING framework, consisting of a *Logic* component and a *Control* component.

We present CARING (Causal and Reliable Reasoning), a modular approach consisting of two components:

- SYMGEN: LLM-based symbolic representation generator (§3.1), which translates problem descriptions into formal symbolic knowledge representations that can be used for symbolic inference.
- SYMINFER: LLM-free symbolic inference engine (§3.2), which performs deliberate reasoning by executing the symbolic representations provided by SYMGEN. With customized Prolog meta-interpreters, SYMINFER supports (i) causal and reliable tracing of the reasoning process (§3.2.1); (ii) various search strategies, such as Depth-First Search (DFS) and Iterative Deepening Search (IDS) (§3.2.2).

The machine-execution nature of SYMINFER guarantees both causality and reliability, as shown in Figure 3. Under the principle of **Causality**, the inference of a new knowledge piece is strictly linked to those existing fragments that are relevant, ensuring precise and limited attribution. This implies that a causal relationship is established only when the preceding event (at the base of the edge) directly influences the subsequent event (at the

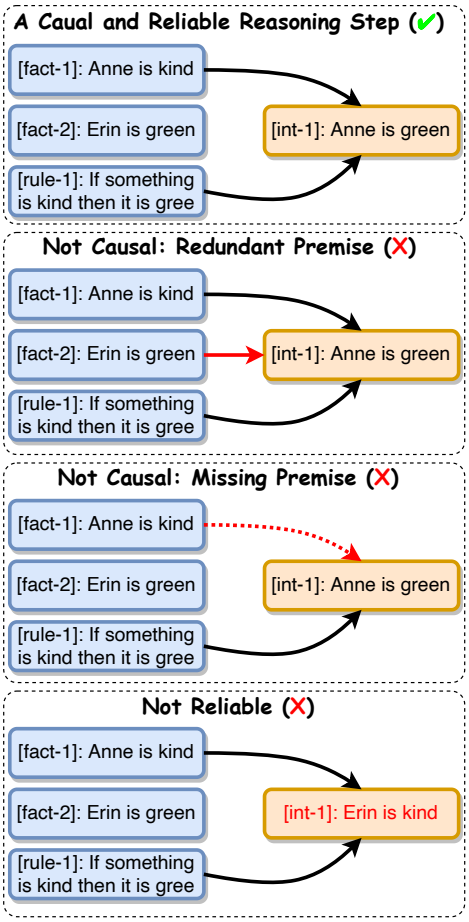


Figure 3: Illustrations of how causality and reliability play important roles in reasoning. LLMs may be (i) non-causal by selecting redundant premises or ignoring relevant ones and (ii) non-reliable by hallucinating erroneous contents.

apex). Regarding **Reliability**, the content within each newly inferred node is the result of a deterministic process, safeguarding it from the kinds of erroneous hallucinations often encountered in outputs from LLMs.

3.1 SymGen: Symbolic Representation Generator

To represent *Logic* (i.e., the knowledge which can be used to solve the problem), we adopt a popular logic programming language, Prolog (Colmerauer and Roussel, 1996). Prolog is a declarative programming language, in which *Logic* is expressed as relations (called Facts and Rules), with several examples shown in Table 1. A computation is initiated by running a query over these relations, which will be further explained in §3.2.

Though LLMs are prone to hallucinate erroneous facts when composing new knowledge (i.e., generating reasoning steps), they are shown to be powerful at understanding natural languages and directly translating them into other formats (Ye and Durrett, 2022; Saparov and He, 2023). Prompting LLMs to translate problem descriptions into symbolic representations just enjoys this strong point of LLMs, as previously shown by Ye et al. (2023). As for implementation, we few-shot prompt LLMs with several human-written in-context demonstrations, each containing a problem and corresponding Prolog representations.

Natural Language	Prolog Code
Fiona is green.	`green(fiona).`
All red, rough things are quiet.	`quiet(X) :- red(X), rough(X).`
Tina makes $18.00 an hour.	`wage(18.00).`
(she is eligible for overtime,) which is paid by your hourly wage + 1/2 your hourly wage.	`overtime_wage(W) :- wage(W1), W is 1.5 * W1.`

Table 1: Examples of problem description snippets and their Prolog representations. It can be seen that the Prolog code is highly declarative, rendering less challenges for LLMs. In other words, the LLM does not need to infer new knowledge, thus avoiding hallucination or planning reasoning paths.

3.2 SymInfer: Symbolic Inference Engine

We use SymInfer to produce answers and reasoning traces by executing the aforementioned symbolic representations. Since we adopt Prolog to represent knowledge, our symbolic inference engine is naturally instantiated with Prolog interpreters. By default, the SWI-Prolog (Wielemaker et al., 2012) interpreter adopts the Depth-First Search (DFS) backtracking strategy and does not yield reasoning proofs. We adopt customized Prolog-based meta-interpreters[1] to achieve two goals: (i) To produce reasoning proofs; (ii) To adopt better search algorithms other than DFS.

3.2.1 Reasoning Tracer

Below is a Prolog meta-interpreter to show the reasoning proofs (Triska, 2024):

```
% Define the operator for proofs
:- op(750, xfy, =>).

% Proof tree generation
mi_tree(true, true).
mi_tree((A,B), (TA,TB)) :-
```

[1]The meta-interpreters used in this work are built upon the implementations of Triska (2024).

```
        mi_tree(A, TA),
        mi_tree(B, TB).
mi_tree(G, builtin(G)) :-
        predicate_property(G, built-in
    ),
        !,
        call(G).
mi_tree(g(G), TBody => G) :-
        mi_clause(G, Body),
        mi_tree(Body, TBody).
```

We showcase how a reasoning trace is induced using the example below. Given a knowledge base like:

```
parent_of(X, Y) :- mother_of(X, Y).
parent_of(X, Y) :- father_of(X, Y).
grandparent_of(X, Y) :-
    parent_of(X, Z), parent_of(Z, Y).
mother_of(morty, beth).
father_of(beth, rick).
```

and a query:

```
?- mi_tree(g(
    grandparent_of(morty, Who)),
    Proof
    ).
```

the output would be

```
Who=rick,
Proof=
(
  (
    (
      true
      =>mother_of(morty, beth)
    )
    =>parent_of(morty, beth),
    (
      true
      =>father_of(beth, rick)
    )
    =>parent_of(beth, rick)
  )
  =>grandparent_of(morty, rick)
).
```

The output proof is ensured to be causal and reliable since it is deterministically generated. A notable advantage is that the reasoning process remains uninfluenced no matter how many distractors are added to the problem description, in contrast to LLMs that can be easily distracted by irrelevant context (Shi et al., 2023).

3.2.2 Search Strategy

The default search strategy of Prolog is DFS, which may lead to infinite loops. For example, given the knowledge base:

```
parent_of(X, Y) :- offspring_of(Y, X).
offspring_of(X, Y) :- parent_of(Y, X).
parent_of(X, Y) :- mother_of(X, Y).
parent_of(X, Y) :- father_of(X, Y).
mother_of(jack, anna).
```

and a query `?- parent_of(jack, Who).` , the backtracking process would repeat over the first two lines without resorting to other lines due to DFS. To address this issue, we adopt Iterative Deepening Search (IDS), in which the backtracking process performs a series of depth-limited searches, each with an increasing depth limit. This leverages the strengths of both Breadth-First Search (BFS) and DFS.

4 Experiments

We briefly introduce our experimental settings in §4.1 and show the experiment results in §4.2.

4.1 Experimental Settings

We present our experimental settings in this section, including our implementation details of the two components (§4.1.1), a brief introduction of the adopted datasets (§4.1.2) and the baselines (§4.1.4).

4.1.1 Implementation

SymGen We adopt the Code-LLaMA (Rozière et al., 2023) family as the base LLMs to translate natural languages into Prolog representations. Our prompting paradigm is in a pure few-shot in-context-learning (ICL) prompting style, without detailed human-written instructions. Each ICL demonstration comprises a question and a piece of Prolog code.

SymInfer We adopt SWI-Prolog (Wielemaker et al., 2012) and PySwip[2] packages to implement the symbolic inference engine. We set the maximum depth to be 20 for Iterative Deepening Search and the number of generated reasoning paths to 20.

4.1.2 Datasets

We evaluate CARING on three popular complex reasoning datasets, including two logical reasoning datasets (ProofWriter (Tafjord et al., 2021) and PrOntoQA (Saparov and He, 2023)) and one arithmetic dataset (GSM8K (Cobbe et al., 2021; Ribeiro et al., 2023)).

ProofWriter ProofWriter (Tafjord et al., 2021) is a commonly-used logical reasoning dataset. It contains many small rulebases of facts and rules, expressed in English. Each rulebase has a set of questions (English statements) that can either be proven true or false using proofs of various depths, or the answer is "Unknown" (in open-world setting,

[2] https://github.com/yuce/pyswip

OWA). The proofs can naturally be represented as directed acyclic graphs (DAGs). The dataset is divided into several sub-sets according to maximum proof depth, namely $\{0, \leq 1, \leq 2, \leq 3, \leq 5\}$. We follow previous work (Pan et al., 2023) to use a 600-instance subset sampled from the most difficult depth-5 test set. We also report additional results on the full depth-5 test set in Appendix §A.1.1.

PrOntoQA PrOntoQA (Saparov and He, 2023) is a synthetic question answering dataset designed for diagnosing the logical reasoning ability of LLMs. Each example aims to validate the feasibility of a statement given a context. We report results on two subsets so we can compare CARING with previous methods. As for the results reported in Table 4, we follow Pan et al. (2023) to adopt the most difficult depth-5 *fictional characters* sub-set, which contains 500 statement-context pairs. As for the results in Table 3, we use the subset adopted by Hao et al. (2023). Similar to ProofWriter, the proofs provided by the dataset can be naturally represented as DAGs.

GSM8K GSM8K (Cobbe et al., 2021) is a multi-step arithmetic reasoning dataset composed of high-quality grade school math word problems. The original GSM8K dataset contains reasoning explanations written in natural language, which raises difficulties in evaluating intermediate steps automatically. Recently, Ribeiro et al. (2023) released a subset that contains 270 questions annotated with structured reasoning proofs in the format of DAGs. We adopt this subset to enable the evaluation of reasoning proofs.

4.1.3 Evaluation Metrics

Ribeiro et al. (2023) proposed two novel metrics to evaluate the quality of the generated reasoning proofs in addition to the prevalent answer accuracy metric. Similar to them, we adopt the following metrics to evaluate both the answers and the generated reasoning proofs.

Answer Accuracy Answer accuracy measures a model's ability to predict the correct answer. A prediction is deemed correct if it is (i) the same as the gold option for multi-choice problems and (ii) the same integer as the gold answer for arithmetic reasoning problems. This metric is the upper bound for other metrics since a reasoning graph would be marked as incorrect without evaluation if the answer is marked as incorrect. We report this metric for all datasets.

	Method	Acc (%)	Proof Sim (%)	
			All	Correct
GPT-4*	CoT	67.41	–	–
	ToT	70.33	–	–
	CR	71.67	–	–
	DetermLR	79.17	–	–
	Logic-LM	79.66	–	–
Code-LLaMA 7B	CoT	46.33	9.69	14.95
	Ours	91.00	72.91	84.39
Code-LLaMA 13B	CoT	46.50	15.69	25.86
	Ours	95.67	80.65	86.00
Code-LLaMA 34B	CoT	52.00	15.76	27.74
	Ours	**96.50**	**81.02**	**86.12**

Table 2: Results on the subset of ProofWriter adopted by Pan et al. (2023). The default setting is 2-shot. "All": on all instances. "Correct": on correctly-predicted instances. *All GPT-4 numbers are from Sun et al. (2023).

Reasoning Proof Similarity As shown in Figure 1, the problems that we are interested in naturally compose reasoning proofs in the format of directed acyclic graphs (DAGs). Reasoning proof similarity $\text{sim}(\mathcal{G}_g, \mathcal{G}_p)$ measures the graph similarity between the gold and the predicted reasoning graphs. We follow Ribeiro et al. (2023) to adopt the graph edit distance function $\delta(\mathcal{G}_g, \mathcal{G}_p)$. This function quantifies the graph edit distance by determining the minimum number of operations required over nodes and edges to transform one graph into the other, thereby enabling a comparison of $\mathcal{G}_g$ and $\mathcal{G}_p$ based on their structural similarities. The reasoning graph similarity is normalized to $[0, 1]$ as:

$$\text{sim}(\mathcal{G}_p, \mathcal{G}_g) = 1 - \frac{\delta(\mathcal{G}_p, \mathcal{G}_g)}{\max\{|N_p| + |E_p|, |N_g| + |E_g|\}} \quad (1)$$

where $|N_p|$ and $|E_p|$ denote the count of nodes and edges, respectively, within the predicted reasoning graph. A similar notation applies to $|N_g|$ and $|E_g|$, which represent the number of nodes and edges in the gold graph. Note that the reasoning graph similarity is set to zero if the predicted answer is incorrect. We report this metric for ProofWriter and GSM8K.

Reasoning Proof Accuracy This metric evaluates the exact match between the gold and the predicted reasoning proofs in terms of both reason-

Base LLM	#Param	#Shot	Method	Acc (%)	Proof Acc (%)	
					All	Correct
LLaMA-1*	33B	8-shot	CoT	87.8	64.8	–
			RAP	94.2	78.8	–
Code-LLaMA	13B	2-shot	CoT	80.2	52.4	53.4
			Ours	**99.0**	**98.2**	**99.2**

Table 3: Results on the PrOntoQA subset that was adopted by RAP (Hao et al., 2023) for comparison with their method. The results marked with * are from their paper.

		Method	Acc (%)	Proof Acc (%)	
			All	Correct	
GPT4*		CoT	98.8	–	–
		Logic-LM	83.2	–	–
Code-LLaMA	7B	CoT	52.0	24.8	28.5
		Ours	98.8	98.4	99.6
	13B	CoT	61.0	32.2	35.9
		Ours	99.4	98.8	99.4
	34B	CoT	82.8	41.0	41.0
		Ours	**100.0**	**100.0**	**100.0**

Table 4: Results on the depth-5 subset of PrOntoQA. The default setting is 2-shot. Results marked with * are GPT-4 results reported by Logic-LM (Pan et al., 2023).

	Method	Acc	Proof Sim (%)	
			All	Correct
7B	CoT	13.70	4.99	36.39
	Ours	12.22	6.57	53.72
13B	CoT	15.56	5.76	37.03
	Ours	21.48	11.66	**54.26**
34B	CoT	35.19	13.04	37.07
	Ours	**42.22**	**22.91**	54.25

Table 5: Results on GSM8K. The default setting is 5-shot.

ing graph structures and textual contents[3]. The reasoning proof accuracy is either 1 or 0 for a single instance, making it a discrete version of reasoning proof similarity. Since this metric requires the dataset to have structured content to enable automatic evaluation, we can only apply it to PrOntoQA, which is specifically designed for easy parsing of the proofs.

4.1.4 Baselines

All baselines prompt the LLMs with few-shot in-context-learning (ICL) demonstrations.

Chain-of-Thought (CoT) CoT prompting (Wei et al., 2022) prompts LLMs with ICL demonstrations that contain both intermediate reasoning steps and answers. It serves as a popular and strong baseline for prompting LLMs to solve problems.

Logic-LM Logic-LM (Pan et al., 2023) is a neuro-symbolic method that adopts symbolic solvers for logical reasoning problems. The main difference between Logic-LM and CARING is: Logic-LM adopts various solvers for multiple datasets and only focuses on answer accuracy, while CARING universally uses one solver (i.e., SWI-Prolog); and more importantly, CARING showcases how SWI-Prolog interpreters can be customized to generate intermediate reasoning proofs and to adopt various search (i.e., problem-solving) strategies.

Search-based Methods We include several search-based methods as baselines. We directly adopt the released results in their papers, since it is too time-consuming to implement these methods on our own. For ProofWriter, we compare our method with GPT-4 based Tree-of-Thoughts (ToT; (Yao et al., 2023)), Cumulative Reasoning (CR; (Zhang et al., 2023)), and DetermLR (Sun et al., 2023). We cannot make comparisons on reasoning proofs because these methods only reported reasoning accuracy. For PrOntoQA, we compare our method with RAP (Hao et al., 2023), in terms of both reasoning accuracy and reasoning proof accuracy.

4.2 Main Results

Tables 2, 3, 4 and 5 show the experimental results on our adopted datasets.

ProofWriter The results on ProofWriter are presented in Table 2. CARING demonstrates notable improvements over existing baselines, particularly in terms of reasoning proof similarity. Utilizing Code-LLaMA-34B, CARING achieves a remark-

[3]Note that our implementation here is simpler than that of Ribeiro et al. (2023) because we only apply this metric to PrOntoQA, which is easy to get evaluated.

able answer accuracy of 96.50% and a reasoning proof similarity of 81.02%, significantly surpassing the most powerful method using GPT-4 that obtains an accuracy of 79.66%.

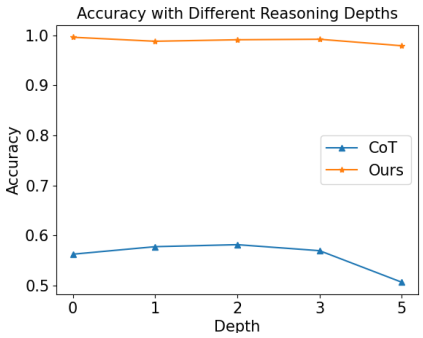
PrOntoQA The results on PrOntoQA are presented in Tables 3 and 4. CARING achieves almost full accuracy with the 13B model. Comparing with RAP, CARING obtains better results in terms of both answer accuracy and proof accuracy even using a smaller base LLM and fewer ICL demonstrations. CARING also outperforms CoT and Logic-LM that use GPT-4.

GSM8K The results on GSM8K are presented in Table 5. This dataset is more challenging than the previous two logical reasoning datasets for CARING, since it is generally believed that symbolic languages are restricted by their limited expressiveness and cannot properly handle the ambiguity in real-world human languages. Surprisingly, with the 34B model, CARING outperforms the strong CoT baseline by a large margin and almost doubles the reasoning proof similarity (22.91% vs. 13.04%). We attribute such improvements to increasingly powerful LLMs, which can correctly translate ambiguous human languages into formal symbolic representations.

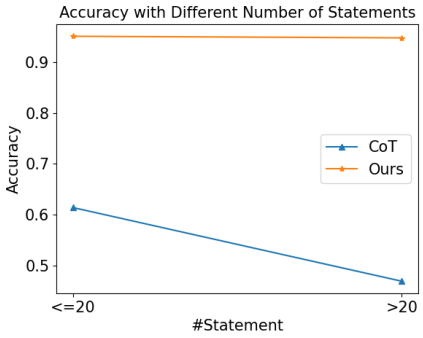
4.3 When Reasoning Becomes More Complex

A key difficulty confronted by reasoning systems is the rapid expansion of possible states as the reasoning process becomes more complex, such as when additional statements are considered or the depth of inference is greater. To investigate how our method handles more complex reasoning problems, we conduct experiments under two controlled settings: (1) **#Depth** ↑: How does the answer accuracy change with #Depth being $<= 0$, $<= 1$, $<= 2$, $<= 3$ and $<= 5$, respectively; (2) **#Statements** ↑: How does the answer accuracy change with #Statements being $<= 20$ and > 20, respectively.

As shown in Figures 4a and 4b, with increasing levels of reasoning intricacy, the answer accuracy of CARING remains steady. In contrast, CoT sees significant decreases in answer accuracy under both settings. This verifies the robustness of CARING against complex reasoning.



(a) Answer accuracy with different reasoning depths.



(b) Answer accuracy with different number of statements.

Figure 4: Answer accuracy when reasoning problems become more complex.

5 Conclusion

This paper presents a framework to address the erroneous reasoning proof problem of LLM-based reasoning systems. Specifically, we develop a neuro-symbolic method called CARING, which produces high-quality reasoning proofs for complex reasoning problems. By implementing customized meta-interpreters for executing Prolog representations and putting LLMs under quarantine during the reasoning phase, CARING ensures the reasoning proofs to be causal and reliable. We conduct experiments on two logical reasoning datasets and one arithmetic reasoning dataset. Experimental results demonstrate our method achieves significant improvements with both final answers and intermediate reasoning proofs. Further analysis indicates CARING remains robust when the reasoning problems become more complex.

Limitations

We observe two limitations regarding our framework:

- The generalization ability of CARING is restricted by the expressiveness of the concerning symbolic representations. In this paper, we showcase a Prolog-based implementation. Other symbolic representations could be explored to generalize CARING to more reason-

ing tasks.

- CARING requires powerful LLMs as symbolic representation generators, which is suggested by the results on GSM8K. This dependence might prevent it from being applied to productions.

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		Acc	Proof Sim	
			All	Correct
7B	Direct	41.78	–	–
	Direct (3-Shot)	43.32	–	–
	CoT	40.95	11.27	17.20
	CoT (3-shot)	42.58	11.52	21.58
	Ours	92.43	75.85	**86.68**
13B	Direct	43.44	–	–
	Direct (3-shot)	44.31	–	–
	CoT	45.88	16.16	27.32
	CoT (3-shot)	54.70	23.18	32.48
	Ours	96.16	80.74	86.34
34B	Direct	44.00	–	–
	Direct (3-shot)	45.93	–	–
	CoT	52.32	15.08	26.30
	CoT (3-shot)	56.50	24.12	34.61
	Ours	**98.11**	**83.17**	85.65

Table 6: Results on ProofWriter. "All" and "Correct" refer to "on all instances" and "on correctly-predicted instances", respectively. "Proof Sim" refers to "Proof Graph Similarity" while "Proof EM" means "Proof Graph Exact Match". The default setting is 2-shot. We additionally conduct 3-shot experiments for baselines to include all types of labels in the in-context demonstrations because this dataset contains three labels: {`true`, `false`, `uncertain`}. We do not conduct 3-shot experiments for our method because it is not sensitive to the number of labels due to its reasoning-by-execution nature.

A Appendix

A.1 Additional Results

A.1.1 Results on ProofWriter

Table 6 shows the results from our implementation on the depth-5 test set of ProofWriter.