

Modal Subordination in Dependent Type Semantics

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Abstract

In the field of natural language processing, the construction of “linguistic pipelines”, which draw on insights from theoretical linguistics, stands in a complementary relationship to the prevailing paradigm of large language models. The rapid development of these pipelines has been fueled by recent advancements, including the emergence of Dependent Type Semantics (DTS) — a type-theoretic framework for natural language semantics. While DTS has been successfully applied to analyze complex linguistic phenomena such as anaphora and presupposition, its capability to account for modal expressions remains an underexplored area. This study aims to address this gap by proposing a framework that extends DTS with modal types. This extension broadens the scope of linguistic phenomena that DTS can account for, including an analysis of modal subordination, where anaphora interacts with modal expressions.

1 Introduction

In recent computational linguistics research, a new approach to natural language processing has seen rapid progress: the use of *linguistic pipelines* (Abzianidze, 2015; Mineshima et al., 2015). These pipelines combine theoretical linguistic insights with computational methods. A key driver of this progress is Dependent Type Semantics (DTS) (Bekki and Mineshima, 2017), a framework for natural language semantics that is rooted in Dependent Type Theory (DTT) (Martin-Löf, 1984). Drawing upon the rich tradition of type theory in programming semantics, DTS provides a compositional framework for the analysis of anaphora and presupposition, which exploits theorem provers in analyzing both anaphora resolution and general inference. By a systematic mapping from formal syntax to semantic interpretation, DTS bridges a significant gap between linguistic theories and computational implementation.

In DTS, the semantic representation (SR) of a sentence corresponds to a type in DTT. The dependency of a type on terms allows reference to terms constructed from the context, thereby reducing both anaphora resolution and presupposition binding to problems of proof search. While DTS provides compelling analyses of complex linguistic phenomena, empirical research on modal expressions remains largely unexplored (but see Tanaka et al. 2015), with existing studies primarily focusing on propositions that abstract away from modal expressions. This study aims to extend DTS by providing an analysis of phenomena involving modal expressions.

Modal expressions, which pertain to the notions of possibility and necessity, have been a central research topic in formal semantics. One of the most discussed phenomena is modal subordination (MS), which, since the pioneering work by Roberts (1989), has been investigated by many researchers (Frank and Kamp, 1997; Kaufmann, 2000; van Rooij, 2005; Asher and McCready, 2007; Keshet and Abney, 2024). (1) and (2) illustrate MS.

- (1) [A wolf]_i might come in. It_i would growl.
- (2) [A wolf]_i might come in. #It_i growls.

As illustrated in (1), an indefinite introduced within the scope of *might* brings a “hypothetical entity” into the discourse¹. To anaphorically refer to this entity, the subsequent discourse must align with the hypothetical scenario in which the entity is assumed to exist, which is typically signaled by the use of *would* in English. The absence of *would*, as demonstrated in (2), blocks this alignment, thereby preventing the pronoun from referring to the indefinite and resulting in a failure of MS.

¹Here, we focus on the analysis of the *de dicto* reading. While example (1) also allows a *de re* reading, where *a wolf* scopes over *might*, a detailed analysis of this reading within DTS is beyond the scope of this paper.

2 Dependent Type Semantics

DTS is a framework developed within the propositions-as-types paradigm. In DTS, the notion of existential quantification $\exists x \in A. B$ is represented by the *dependent product types* $(x : A) \times B$, which are types of pairs (a, b) such that a is of type A and b is of type $B(a)$. The SR of the unmodalized sentence in (3a) is given in (3b). We employ vertical notation for the dependent product type in the subsequent discussion, and π_1 denotes the proof constructor that yields the first projection of such a pair.

- (3) a. A wolf came in.
 b.
$$\left[\begin{array}{c} u : \left[\begin{array}{c} x : \text{entity} \\ \text{wolf}(x) \end{array} \right] \\ \text{comeIn}(\pi_1(u)) \end{array} \right]$$

Bekki (2023) analyzes pronouns as introducing *underspecified types*, written as $(x@A) \times B$. Here, the variable x functions as a placeholder that is to be replaced by a proof of type A from a given context. Example (4) briefly illustrates how anaphora resolution proceeds in DTS.

- (4) [A wolf]_i came in. It_i growled.
 a.
$$\left[\begin{array}{c} v : \left[\begin{array}{c} u : \left[\begin{array}{c} x : \text{entity} \\ \text{wolf}(x) \end{array} \right] \\ \text{comeIn}(\pi_1(u)) \end{array} \right] \\ w@ \left[\begin{array}{c} z : \text{entity} \\ \neg \text{human}(z) \end{array} \right] \\ \text{growl}(\pi_1(w)) \end{array} \right]$$

 b.
$$\left[\begin{array}{c} v : \left[\begin{array}{c} u : \left[\begin{array}{c} x : \text{entity} \\ \text{wolf}(x) \end{array} \right] \\ \text{comeIn}(\pi_1(u)) \end{array} \right] \\ \text{growl}(\pi_1 \pi_1(v)) \end{array} \right]$$

The underspecified type in (4a) is eliminated through *type-checking*, a process that validates whether an SR is a well-formed type under a given context. Upon encountering $w@((z : \text{entity}) \times \neg \text{human}(z))$, the type-checking algorithm attempts to find a proof of type $(z : \text{entity}) \times \neg \text{human}(z)$. In this specific case, such a proof is successfully found and substituted for the variable x . Subsequently, $\pi_1 \pi_1(v)$, which corresponds to the entity x (i.e., the first element of $\pi_1(v)$), serves as the argument of the predicate **growl**, thereby resolving the pronoun *it*.

3 Modal DTS

To account for modal expressions within DTS, we propose Modal DTS, an extension grounded in Contextual Modal Type Theory (CMTT) (Nanevski

et al., 2008). Modal DTS introduces two novel type constructors: $[\Psi]$ for necessity and $\langle \Psi \rangle$ for possibility, both of which are parameterized by a context Ψ . In a manner analogous to possible worlds semantics, Ψ serves as a proxy for a domain of possible worlds; accordingly, $[\Psi]$ and $\langle \Psi \rangle$ indicate that the propositions within their scope hold in all or some worlds, respectively, where Ψ is true. As an example, Figure 1 illustrates the SR of (1).

$$\left[\begin{array}{c} v : \langle \Psi \rangle \left[\begin{array}{c} u : \left[\begin{array}{c} x : \text{entity} \\ \text{wolf}(x) \end{array} \right] \\ \text{comeIn}(\pi_1(u)) \end{array} \right] \\ [\Psi] \left[\begin{array}{c} w@ \left[\begin{array}{c} z : \text{entity} \\ \neg \text{human}(z) \end{array} \right] \\ \text{growl}(\pi_1(w)) \end{array} \right] \end{array} \right]$$

Figure 1: SR of (1) before anaphora resolution

As described in § 2, a dependent product type is a type of pairs where the second conjunct depends on the first element of the pair, i.e., the second conjunct is within the scope of the dependent product type. Accordingly, in Figure 1, where the SR of the first sentence forms the first conjunct, and that of the second sentence the second conjunct, the continuation *it would growl* quantifies over the subset of possible worlds in which *a wolf came in*.

3.1 Contextual Modal Type Theory

Intuitionistic modal logic for necessity is founded on the judgmental notion of categorical truth. Nanevski et al. (2008) examined the consequences of relativizing these notions of categorical truth to explicitly specified contexts, resulting in the formulation of contextual modal logic and its type-theoretic counterpart. Nanevski et al. (2008) advanced the structural approach to intuitionistic modal logic by allowing arbitrary contexts to be internalized within propositions. From a type-theoretic standpoint, CMTT is based on contextual modal logic and provides formal definitions for proof term assignment, substitution on terms, proof reductions and expansions, as well as strong normalization. From a logical standpoint, CMTT constitutes a relativized variant of the intuitionistic modal logic S4.

Modal DTS is a framework that uniquely integrates the dependent types of DTS with the modal types of CMTT. The newly introduced types are grounded in the notions of *contextual necessity* and *contextual possibility* as defined in CMTT. Contextual necessity, denoted as $[\Psi]A$, indicates that

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