

pair2vec: Compositional Word-Pair Embeddings for Cross-Sentence Inference

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Abstract

Reasoning about implied relationships (e.g. paraphrastic, common sense, encyclopedic) between pairs of words is crucial for many cross-sentence inference problems. This paper proposes new methods for learning and using embeddings of word *pairs* that implicitly represent background knowledge about such relationships. Our pairwise embeddings are computed as a compositional function on word representations, which is learned by maximizing the pointwise mutual information (PMI) with the contexts in which the two words co-occur. We add these representations to the cross-sentence attention layer of existing inference models (e.g. BiDAF for QA, ESIM for NLI), instead of extending or replacing existing word embeddings. Experiments show a gain of 2.7% on the recently released SQuAD 2.0 and 1.3% on MultiNLI. Our representations also aid in better generalization with gains of around 6-7% on adversarial SQuAD datasets, and 8.8% on the adversarial entailment test set by Glockner et al. (2018).

1 Introduction

Reasoning about relationships between pairs of words is crucial for cross sentence inference problems such as question answering (QA) and natural language inference (NLI). In NLI, for example, given the premise “*golf is prohibitively expensive*”, inferring that the hypothesis “*golf is a cheap pastime*” is a contradiction requires one to know that *expensive* and *cheap* are antonyms. Recent work (Glockner et al., 2018) has shown that current models, which rely heavily on unsupervised single-word embeddings, struggle to learn such relationships. In this paper, we show that they can be learned with word *pair* vectors (pair2vec¹),

¹<https://github.com/mandarjoshi90/pair2vec>

| X | Y | Contexts |
|-----------------|---------------|--|
| <i>hot</i> | <i>cold</i> | with X and Y baths too X or too Y neither X nor Y |
| <i>Portland</i> | <i>Oregon</i> | in X, Y the X metropolitan area in Y X International Airport in Y |
| <i>crop</i> | <i>wheat</i> | food X are maize, Y, etc dry X, such as Y, more X circles appeared in Y fields |
| <i>Android</i> | <i>Google</i> | X OS comes with Y play the X team at Y X is developed by Y |

Table 1: Example word pairs and their contexts.

which are trained unsupervised, and which significantly improve performance when added to existing cross-sentence attention mechanisms.

Unlike single-word representations, which typically model the co-occurrence of a target word x with its context c , our word-pair representations are learned by modeling the three-way co-occurrence between words (x, y) and the context c that ties them together, as seen in Table 1. While similar training signals have been used to learn models for ontology construction (Hearst, 1992; Snow et al., 2005; Turney, 2005; Shwartz et al., 2016) and knowledge base completion (Riedel et al., 2013), this paper shows, for the first time, that large scale learning of pairwise embeddings can be used to directly improve the performance of neural cross-sentence inference models.

More specifically, we train a feedforward network $R(x, y)$ that learns representations for the individual words x and y , as well as how to compose them into a single vector. Training is done by maximizing a generalized notion of the pointwise mutual information (PMI) among x , y , and their context c using a variant of negative sampling (Mikolov et al., 2013a). Making $R(x, y)$ a

compositional function on individual words alleviates the sparsity that necessarily comes with embedding pairs of words, even at a very large scale.

We show that our embeddings can be added to existing cross-sentence inference models, such as BiDAF++ (Seo et al., 2017; Clark and Gardner, 2018) for QA and ESIM (Chen et al., 2017) for NLI. Instead of changing the word embeddings that are fed into the encoder, we add the pretrained *pair* representations to *higher layers* in the network where cross sentence attention mechanisms are used. This allows the model to use the background knowledge that the pair embeddings implicitly encode to reason about the likely relationships between the pairs of words it aligns.

Experiments show that simply adding our word-pair embeddings to existing high-performing models, which already use ELMo (Peters et al., 2018), results in sizable gains. We show 2.72 F1 points over the BiDAF++ model (Clark and Gardner, 2018) on SQuAD 2.0 (Rajpurkar et al., 2018), as well as a 1.3 point gain over ESIM (Chen et al., 2017) on MultiNLI (Williams et al., 2018). Additionally, our approach generalizes well to adversarial examples, with a 6-7% F1 increase on adversarial SQuAD (Jia and Liang, 2017) and a 8.8% gain on the Glockner et al. (2018) NLI benchmark. An analysis of `pair2vec` on word analogies suggests that it complements the information in single-word representations, especially for encyclopedic and lexicographic relations.

2 Unsupervised Pretraining

Extending the distributional hypothesis to word pairs, we posit that similar word *pairs* tend to occur in similar contexts, and that the contexts provide strong clues about the likely relationships that hold between the words (see Table 1). We assume a dataset of (x, y, c) triplets, where each instance depicts a word pair (x, y) and the context c in which they appeared. We learn two compositional representation functions, $R(x, y)$ and $C(c)$, to encode the pair and the context, respectively, as d -dimensional vectors (Section 2.1). The functions are trained using a variant of negative sampling, which tries to embed word pairs (x, y) close to the contexts c with which they appeared (Section 2.2).

2.1 Representation

Our word-pair and context representations are both fixed-length vectors, composed from individ-

ual words. The word-pair representation function $R(x, y)$ first embeds and normalizes the individual words with a shared lookup matrix E_a :

$$\mathbf{x} = \frac{E_a(x)}{\|E_a(x)\|} \quad \mathbf{y} = \frac{E_a(y)}{\|E_a(y)\|}$$

These vectors, along with their element-wise product, are fed into a four-layer perceptron:

$$R(x, y) = MLP^4(\mathbf{x}, \mathbf{y}, \mathbf{x} \circ \mathbf{y})$$

The context $c = c_1 \dots c_n$ is encoded as a d -dimensional vector using the function $C(c)$. $C(c)$ embeds each token c_i with a lookup matrix E_c , contextualizes it with a single-layer Bi-LSTM, and then aggregates the entire context with attentive pooling:

$$\begin{aligned} \mathbf{c}_i &= E_c(c_i) \\ \mathbf{h}_1 \dots \mathbf{h}_n &= \text{BiLSTM}(\mathbf{c}_1 \dots \mathbf{c}_n) \\ w &= \text{softmax}_i(\mathbf{k}\mathbf{h}_i) \\ C(c) &= \sum_i w_i \mathbf{W}\mathbf{h}_i \end{aligned}$$

where $\mathbf{W} \in \mathbb{R}^{d \times d}$ and $\mathbf{k} \in \mathbb{R}^d$. All parameters, including the lookup tables E_a and E_c , are trained.

Our representation is similar to two recently-proposed frameworks by Washio and Kato (2018a,b), but differs in that: (1) they use dependency paths as context, while we use surface form; (2) they encode the context as either a lookup table or the last state of a unidirectional LSTM. We also use a different objective, which we discuss next.

2.2 Objective

To optimize our representation functions, we consider two variants of negative sampling (Mikolov et al., 2013a): bivariate and multivariate. The original bivariate objective models the two-way distribution of context and (monolithic) word pair co-occurrences, while our multivariate extension models the three-way distribution of word-word-context co-occurrences. We further augment the multivariate objective with typed sampling to up-sample harder negative examples. We discuss the impact of the bivariate and multivariate objectives (and other components) in Section 4.3.

Bivariate Negative Sampling Our objective aspires to make $R(x, y)$ and $C(c)$ similar (have high inner products) for (x, y, c) that were observed together in the data. At the same time, we wish

| | |
|---------------------|---|
| Bivariate | $J_{2NS}(x, y, c) = \log \sigma(R(x, y) \cdot C(c)) + \sum_{i=1}^{k_c} \log \sigma(-R(x, y) \cdot C(c_i^N))$ |
| Multivariate | $J_{3NS}(x, y, c) = J_{2NS}(x, y, c) + \sum_{i=1}^{k_x} \log \sigma(-R(x_i^N, y) \cdot C(c)) + \sum_{i=1}^{k_y} \log \sigma(-R(x, y_i^N) \cdot C(c))$ |

Table 2: The bivariate and multivariate negative sampling objectives. The superscript N marks randomly sampled components, with k_* being the negative sample size per instance. The equations present per-instance objectives.

to keep our pair vectors *dis*-similar from random context vectors. In a straightforward application of the original (bivariate) negative sampling objective, we could generate a negative example from each observed (x, y, c) instance by replacing the original context c with a randomly-sampled context c^N (Table 2, J_{2NS}).

Assuming that the negative contexts are sampled from the empirical distribution $P(\cdot, \cdot, c)$ (with $P(x, y, c)$ being the portion of (x, y, c) instances in the dataset), we can follow Levy and Goldberg (2014) to show that this objective converges into the pointwise mutual information (PMI) between the word pair and the context.

$$R(x, y) \cdot C(c) = \log \frac{P(x, y, c)}{k_c P(x, y, \cdot) P(\cdot, \cdot, c)}$$

This objective mainly captures co-occurrences of monolithic pairs and contexts, and is limited by the fact that the training data, by construction, only contains pairs occurring within a sentence. For better generalization to cross-sentence tasks, where the pair distribution differs from that of the training data, we need a multivariate objective that captures the full three-way (x, y, c) interaction.

Multivariate Negative Sampling We introduce negative sampling of target words, x and y , in addition to negative sampling of contexts c (Table 2, J_{3NS}). Our new objective also converges to a novel multivariate generalization of PMI, different from previous PMI extensions that were inspired by information theory (Van de Cruys, 2011) and heuristics (Jameel et al., 2018).² Following Levy and Goldberg (2014), we can show that when replacing target words in addition to contexts, our objective will converge³ to the optimal value in Equation 1:

$$R(x, y) \cdot C(c) = \log \frac{P(x, y, c)}{Z_{x, y, c}} \quad (1)$$

²See supplementary material for their exact formulations.

³A full proof is provided in the supplementary material.

where $Z_{x, y, c}$, the denominator, is:

$$\begin{aligned} Z_{x, y, c} &= k_c P(\cdot, \cdot, c) P(x, y, \cdot) \\ &\quad + k_x P(x, \cdot, \cdot) P(\cdot, y, c) \\ &\quad + k_y P(\cdot, y, \cdot) P(x, \cdot, c) \end{aligned} \quad (2)$$

This optimal value deviates from previous generalizations of PMI by having a linear mixture of marginal probability products in its denominator. By introducing terms such as $P(x, \cdot, c)$ and $P(\cdot, y, c)$, the objective penalizes spurious correlations between words and contexts that disregard the other target word. For example, it would assign the pattern “ X is a Y ” a high score with $(banana, fruit)$, but a lower score with $(cat, fruit)$.

Typed Sampling In multivariate negative sampling, we typically replace x and y by sampling from their unigram distributions. In addition to this, we also sample uniformly from the top 100 words according to cosine similarity using distributional word vectors. This is done to encourage the model to learn relations between specific instances as opposed to more general types. For example, using *California* as a negative sample for *Oregon* helps the model to learn that the pattern “ X is located in Y ” fits the pair $(Portland, Oregon)$, but not the pair $(Portland, California)$. Similar adversarial constraints were used in knowledge base completion (Toutanova et al., 2015) and word embeddings (Li et al., 2017).⁴

3 Integrating pair2vec into Models

We first present a general outline for incorporating `pair2vec` into attention-based architectures, and then discuss changes made to BiDAF++ and ESIM. The key idea is to inject our pairwise representations into the attention layer by reusing the cross-sentence attention weights. In addition to attentive pooling over single word representations, we also pool over cross-sentence word pair embeddings (Figure 1).

⁴Applying typed sampling also changes the value to which our objective will converge, and will replace the unigram probabilities in Equation (2) to reflect the type-based distribution.

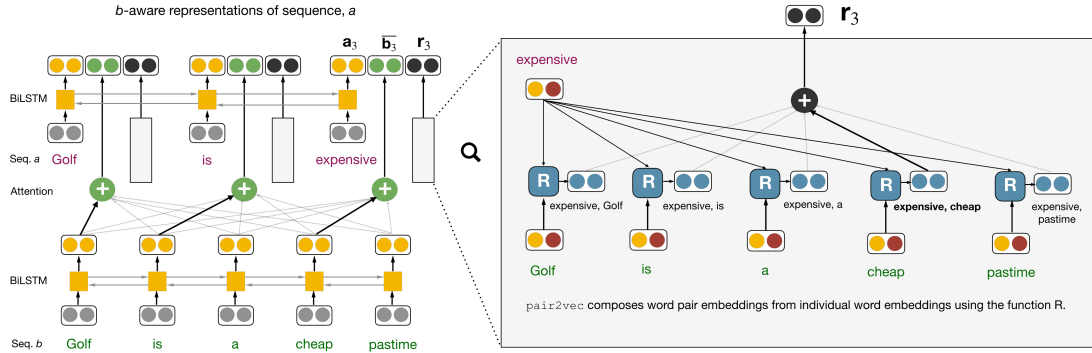


Figure 1: A typical architecture of a cross-sentence inference model (left), and how `pair2vec` is added to it (right). Given two sequences, a and b , existing models create b -aware representations of words in a . For any word a_i , this typically involves the BiLSTM representation of word a_i (\mathbf{a}_i), and an attention-weighted sum over b 's BiLSTM states with a_i as the query (\mathbf{b}_i). To these, we add the word-pair representation of a_i and each word in b , weighted by attention (\mathbf{r}_i). Thicker attention arrows indicate stronger word pair alignments (e.g. *cheap, expensive*).

3.1 General Approach

Pair Representation We assume that we are given two sequences $a = a_1 \dots a_n$ and $b = b_1 \dots b_m$. We represent the word-pair embeddings between a and b using the pretrained `pair2vec` model as:

$$\mathbf{r}_{i,j} = \left[\frac{R(a_i, b_j)}{\|R(a_i, b_j)\|}; \frac{R(b_j, a_i)}{\|R(b_j, a_i)\|} \right] \quad (3)$$

We include embeddings in both directions, $R(a_i, b_j)$ and $R(b_j, a_i)$, because the many relations can be expressed in both directions; e.g., hypernymy can be expressed via “ X is a type of Y ” as well as “ Y such as X ”. We take the L_2 normalization of each direction’s pair embedding because the heavy-tailed distribution of word pairs results in significant variance of their norms.

Base Model Let $\mathbf{a}_1 \dots \mathbf{a}_n$ and $\mathbf{b}_1 \dots \mathbf{b}_m$ be the vector representations of sequences a and b , as produced by the input encoder (e.g. ELMo embeddings contextualized with model-specific BiLSTMs). Furthermore, we assume that the base model computes soft word alignments between a and b via co-attention (4, 5), which are then used to compute b -aware representations of a :

$$s_{i,j} = f_{att}(\mathbf{a}_i, \mathbf{b}_j) \quad (4)$$

$$\alpha = \text{softmax}_j(s_{i,j}) \quad (5)$$

$$\bar{\mathbf{b}}_i = \sum_{j=0}^m \alpha_{i,j} \mathbf{b}_j \quad (6)$$

$$\mathbf{a}_i^{inf} = [\mathbf{a}_i; \bar{\mathbf{b}}_i] \quad (7)$$

The symmetric term \mathbf{b}_j^{inf} is defined analogously. We refer to \mathbf{a}^{inf} and \mathbf{b}^{inf} as the inputs to the *infer-*

ence layer, since this layer computes some function over aligned word pairs, typically via a feed-forward network and LSTMs. The inference layer is followed by aggregation and output layers.

Injecting `pair2vec` We conjecture that the inference layer effectively learns word-pair relationships from training data, and it should, therefore, help to augment its input with `pair2vec`. We augment \mathbf{a}_i^{inf} (7) with the pair vectors $\mathbf{r}_{i,j}$ (3) by concatenating a weighted average of the pair vectors $r_{i,j}$ involving a_i , where the weights are the same $\alpha_{i,j}$ computed via attention in (5):

$$\mathbf{r}_i = \sum_j \alpha_{i,j} \mathbf{r}_{i,j} \quad (8)$$

$$\mathbf{a}_i^{inf} = [\mathbf{a}_i; \bar{\mathbf{b}}_i; \mathbf{r}_i] \quad (9)$$

The symmetric term \mathbf{b}_j^{inf} is defined analogously.

3.2 Question Answering

We augment the inference layer in the BiDAF++ model with `pair2vec`. BiDAF++ is an improved version of the BiDAFNoAnswer (Seo et al., 2017; Levy et al., 2017) which includes self-attention and ELMo embeddings from Peters et al. (2018). We found this variant to be stronger than the baselines presented in Rajpurkar et al. (2018) by over 2.5 F1. We use BiDAF++ as a baseline since its architecture is typical for QA systems, and, until recently, was state-of-the-art on SQuAD 2.0 and other benchmarks.

BiDAF++ Let \mathbf{a} and \mathbf{b} be the outputs of the passage and question encoders respectively (in place of the standard \mathbf{p} and \mathbf{q} notations). The inference layer’s inputs \mathbf{a}_i^{inf} are defined similarly to

the generic model’s in (7), but also contain an aggregation of the elements in \mathbf{a} , with better-aligned elements receiving larger weights:

$$\mu = \text{softmax}_i(\max_j s_{i,j}) \quad (10)$$

$$\hat{\mathbf{a}}_i = \sum_i \mu_i \mathbf{a}_i \quad (11)$$

$$\mathbf{a}_i^{inf} = [\mathbf{a}_i; \bar{\mathbf{b}}_i; \mathbf{a}_i \circ \bar{\mathbf{b}}_i; \hat{\mathbf{a}}] \quad (12)$$

In the later layers, \mathbf{a}^{inf} is recontextualized using a BiGRU and self attention. Finally a prediction layer predicts the start and end tokens.

BiDAF++ with pair2vec To add our pair vectors, we simply concatenate \mathbf{r}_i (3) to \mathbf{a}_i^{inf} (12):

$$\mathbf{a}_i^{inf} = [\mathbf{a}_i; \bar{\mathbf{b}}_i; \mathbf{a}_i \circ \bar{\mathbf{b}}_i; \hat{\mathbf{a}}; \mathbf{r}_i] \quad (13)$$

3.3 Natural Language Inference

For NLI, we augment the ESIM model (Chen et al., 2017), which was previously state-of-the-art on both SNLI (Bowman et al., 2015) and MultiNLI (Williams et al., 2018) benchmarks.

ESIM Let \mathbf{a} and \mathbf{b} be the outputs of the premise and hypothesis encoders respectively (in place of the standard \mathbf{p} and \mathbf{h} notations). The inference layer’s inputs \mathbf{a}_i^{inf} (and \mathbf{b}_j^{inf}) are defined similarly to the generic model’s in (7):

$$\mathbf{a}_i^{inf} = [\mathbf{a}_i; \bar{\mathbf{b}}_i; \mathbf{a}_i \circ \bar{\mathbf{b}}_i; \mathbf{a}_i - \bar{\mathbf{b}}_i] \quad (14)$$

In the later layers, \mathbf{a}^{inf} and \mathbf{b}^{inf} are projected, recontextualized, and converted to a fixed-length vector for each sentence using multiple pooling schemes. These vectors are then passed on to an output layer, which predicts the class.

ESIM with pair2vec To add our pair vectors, we simply concatenate \mathbf{r}_i (3) to \mathbf{a}_i^{inf} (14):

$$\mathbf{a}_i^{inf} = [\mathbf{a}_i; \bar{\mathbf{b}}_i; \mathbf{a}_i \circ \bar{\mathbf{b}}_i; \mathbf{a}_i - \bar{\mathbf{b}}_i; \mathbf{r}_i] \quad (15)$$

A similar augmentation of ESIM was recently proposed in KIM (Chen et al., 2018). However, their pair vectors are composed of WordNet features, while our pair embeddings are learned directly from text (see further discussion in Section 6).

4 Experiments

For experiments on QA (Section 4.1) and NLI (Section 4.2), we use our full model which includes multivariate and typed negative sampling. We discuss ablations in Section 4.3

| Benchmark | | BiDAF | +pair2vec | Δ |
|------------|----|-------|-----------|----------|
| SQuAD 2.0 | EM | 65.66 | 68.02 | +2.36 |
| | F1 | 68.86 | 71.58 | +2.72 |
| AddSent | EM | 37.50 | 44.20 | +6.70 |
| | F1 | 42.55 | 49.69 | +7.14 |
| AddOneSent | EM | 48.20 | 53.30 | +5.10 |
| | F1 | 54.02 | 60.13 | +6.11 |

Table 3: Performance on SQuAD 2.0 and adversarial SQuAD (AddSent and AddOneSent) benchmarks, with and without pair2vec. All models have ELMo.

| Benchmark | ESIM | +pair2vec | Δ |
|------------|-------|-----------|----------|
| Matched | 79.68 | 81.03 | +1.35 |
| Mismatched | 78.80 | 80.12 | +1.32 |

Table 4: Performance on MultiNLI, with and without pair2vec. All models have ELMo.

Data We use the January 2018 dump of English Wikipedia, containing 96M sentences to train pair2vec. We restrict the vocabulary to the 100K most frequent words. Preprocessing removes all out-of-vocabulary words in the corpus. We consider each word pair within a window of 5 in the preprocessed corpus, and subsample⁵ instances based on pair probability with a threshold of $5 \cdot 10^{-7}$. We define the context as one word each to the left and right, and all the words in between each pair, replacing both target words with placeholders X and Y (see Table 1). More details can be found in the supplementary material.

4.1 Question Answering

We experiment on the SQuAD 2.0 QA benchmark (Rajpurkar et al., 2018), as well as the adversarial datasets of SQuAD 1.1 (Rajpurkar et al., 2016; Jia and Liang, 2017). Table 3 shows the performance of BiDAF++, with ELMo, before and after adding pair2vec. Experiments on SQuAD 2.0 show that our pair representations improve performance by 2.72 F1. Moreover, adding pair2vec also results in better generalization on the adversarial SQuAD datasets with gains of 7.14 and 6.11 F1.

4.2 Natural Language Inference

We report the performance of our model on MultiNLI and the adversarial test set from Glockner et al. (2018) in Table 5. We outperform the

⁵Like in word2vec, subsampling reduces the size of the dataset and speeds up training. For this, we define the word pair probability as the product of unigram probabilities.

| Model | Accuracy |
|----------------------------|-------------|
| Rule-based Models | |
| WordNet Baseline | 85.8 |
| Models with GloVe | |
| ESIM (Chen et al., 2017) | 77.0 |
| KIM (Chen et al., 2018) | 87.7 |
| ESIM + pair2vec | 92.9 |
| Models with ELMo | |
| ESIM (Peters et al., 2018) | 84.6 |
| ESIM + pair2vec | 93.4 |

Table 5: Performance on the adversarial NLI test set of Glockner et al. (2018).

| Model | EM (Δ) | F1 (Δ) |
|-------------------------|-----------------|-----------------|
| pair2vec (Full Model) | 69.20 | 72.68 |
| Composition: 2 Layers | 68.35 (-0.85) | 71.65 (-1.03) |
| Composition: Multiply | 67.10 (-2.20) | 70.20 (-2.48) |
| Objective: Bivariate NS | 68.63 (-0.57) | 71.98 (-0.70) |
| Unsupervised: Pair Dist | 67.07 (-2.13) | 70.24 (-2.44) |
| No pair2vec (BiDAF) | 66.66 (-2.54) | 69.90 (-2.78) |

Table 6: Ablations on the Squad 2.0 development set show that argument sampling as well as using a deeper composition function are useful.

ESIM + ELMo baseline by 1.3% on the matched and mismatched portions of the dataset.

We also record a gain of 8.8% absolute over ESIM on the Glockner et al. (2018) dataset, setting a new state of the art. Following standard practice (Glockner et al., 2018), we train all models on a combination of SNLI (Bowman et al., 2015) and MultiNLI. Glockner et al. (2018) show that with the exception of KIM (Chen et al., 2018), which uses WordNet features, several NLI models fail to generalize to this setting which involves lexical inference. For a fair comparison with KIM on the Glockner test set, we replace ELMo with GloVe embeddings, and still outperform KIM by almost halving the error rate.

4.3 Ablations

Ablating parts of pair2vec shows that all components of the model (Section 2) are useful. We ablate each component and report the EM and F1 on the development set of SQuAD 2.0 (Table 6). The full model, which uses a 4-layer MLP for $R(x, y)$ and trains with multivariate negative sampling, achieves the highest F1 of 72.68.

We experiment with two alternative composition functions, a 2-layer MLP (*Composition: 2 Layers*) and element-wise multiplication (*Compo-*

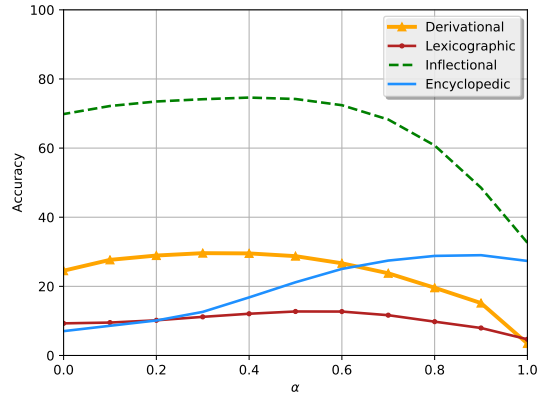


Figure 2: Accuracy as a function of the interpolation parameter α (see Eq. (16)). The $\alpha=0$ configuration relies only on fastText (Bojanowski et al., 2017), while $\alpha=1$ reflects pair2vec.

sition: *Multiply*), which yield significantly smaller gains over the baseline BiDAF++ model. This demonstrates the need for a deep composition function. Eliminating sampling of target words (x, y) from the objective (*Objective: Bivariate NS*) results in a drop of 0.7 F1, accounting for about a quarter of the overall gain. This suggests that while the bulk of the signal is mined from the pair-context interactions, there is also valuable information in other interactions as well.

We also test whether specific pre-training of word pair representations is useful by replacing pair2vec embeddings with the vector offsets of pre-trained word embeddings (*Unsupervised: Pair Dist*). We follow the PairDistance method for word analogies (Mikolov et al., 2013b), and represent the pair (x, y) as the L2 normalized difference of single-word vectors: $(x - y) / \|x - y\|$. We use the same fastText (Bojanowski et al., 2017) word vectors with which we initialized pair2vec before training. We observe a gain of only 0.34 F1 over the baseline.

5 Analysis

In Section 4, we showed that pair2vec adds information complementary to single-word representations like ELMo. Here, we ask what this extra information is, and try to characterize which word relations are better captured by pair2vec. To that end, we evaluate performance on a word analogy dataset with over 40 different relation types (Section 5.1), and observe how pair2vec fills hand-crafted relation patterns (Section 5.2).

| Relation | 3CosAdd | +pair2vec | α^* |
|-------------------|---------|-----------|------------|
| Country:Capital | 1.2 | 86.1 | 0.9 |
| Name:Occupation | 1.8 | 44.6 | 0.8 |
| Name:Nationality | 0.1 | 42.0 | 0.9 |
| UK City:County | 0.7 | 31.7 | 1.0 |
| Country:Language | 4.0 | 28.4 | 0.8 |
| Verb 3pSg:Ved | 49.1 | 61.7 | 0.6 |
| Verb Ving:Ved | 61.1 | 73.3 | 0.5 |
| Verb Inf:Ved | 58.5 | 70.1 | 0.5 |
| Noun+less | 4.8 | 16.0 | 0.2 |
| Substance Meronym | 3.8 | 14.5 | 0.6 |

Table 7: The top 10 analogy relations for which interpolating with `pair2vec` improves performance. α^* is the optimal interpolation parameter for each relation.

5.1 Quantitative Analysis: Word Analogies

Word Analogy Dataset Given a word pair (a, b) and word x , the word analogy task involves predicting a word y such that $a : b :: x : y$. We use the Bigger Analogy Test Set (BATS, Gladkova et al., 2016) which contains four groups of relations: encyclopedic semantics (e.g., person-profession as in *Einstein-physicist*), lexicographic semantics (e.g., antonymy as in *cheap-expensive*), derivational morphology (e.g., noun forms as in *oblige-obligation*), and inflectional morphology (e.g., noun-plural as in *bird-birds*). Each group contains 10 sub-relations.

Method We interpolate `pair2vec` and `3CosAdd` (Mikolov et al., 2013b; Levy et al., 2014) scores on `fastText` embeddings, as follows:

$$\text{score}(y) = \alpha \cdot \cos(\mathbf{r}_{a,b}, \mathbf{r}_{x,y}) + (1 - \alpha) \cdot \cos(\mathbf{b} - \mathbf{a} + \mathbf{x}, \mathbf{y}) \quad (16)$$

where \mathbf{a} , \mathbf{b} , \mathbf{x} , and \mathbf{y} represent `fastText` embeddings⁶ and $\mathbf{r}_{a,b}$, $\mathbf{r}_{x,y}$ represent the `pair2vec` embedding for the word pairs (a, b) and (x, y) , respectively; α is the linear interpolation parameter. Following prior work (Mikolov et al., 2013b), we return the highest-scoring y in the entire vocabulary, excluding the given words a , b , and x .

Results Figure 2 shows how the accuracy on each category of relations varies with α . For all four groups, adding `pair2vec` to `3CosAdd` results in significant gains. In particular, the biggest relative improvements are observed for encyclopedic (356%) and lexicographic (51%) relations.

⁶The `fastText` embeddings in the analysis were retrained using the same Wikipedia corpus used to train `pair2vec` to control for the corpus when comparing the two methods.

Table 7 shows the specific relations in which `pair2vec` made the largest absolute impact. The gains are particularly significant for relations where `fastText` embeddings provide limited signal. For example, the accuracy for *substance meronyms* goes from 3.8% to 14.5%. In some cases, there is also a synergistic effect; for instance, in *noun+less*, `pair2vec` alone scored 0% accuracy, but mixing it with `3CosAdd`, which got 4.8% on its own, yielded 16% accuracy.

These results, alongside our experiments in Section 4, strongly suggest that `pair2vec` encodes information complementary to that in single-word embedding methods such as `fastText` and `ELMo`.

5.2 Qualitative Analysis: Slot Filling

To further explore how `pair2vec` encodes such complementary information, we consider a setting similar to that of knowledge base completion: given a Hearst-like context pattern c and a single word x , predict the other word y from the entire vocabulary. We rank candidate words y based on the scoring function in our training objective: $R(x, y) \cdot C(c)$. We use a fixed set of example relations and manually define their predictive context patterns and a small set of candidate words x .

Table 8 shows the top three y words. The model embeds (x, y) pairs close to contexts that reflect their relationship. For example, substituting *Portland* in the city-state pattern (“*in X, Y.*”), the top two words are *Oregon* and *Maine*, both US states with cities named Portland. When used with the city-city pattern (“*from X to Y.*”), the top two words are *Salem* and *Astoria*, both cities in Oregon. The word-context interaction often captures multiple relations; for example, *Monet* is used to refer to the painter (*profession*) as well as his paintings.

As intended, `pair2vec` captures the three-way word-word-context interaction, and not just the two-way word-context interaction (as in single-word embeddings). This profound difference allows `pair2vec` to complement single-word embeddings with additional information.

6 Related Work

Pretrained Word Embeddings Many state-of-the-art models initialize their word representations using pretrained embeddings such as `word2vec` (Mikolov et al., 2013a) or `ELMo` (Peters et al., 2018). These representations are typically trained using an interpretation of the Distributional Hy-

| Relation | Context | X | Y (Top 3) |
|--------------------|-------------------------|--------------------|--|
| Antonymy/Exclusion | either X or Y | accept hard | <i>reject, refuse, recognise</i> <i>soft, brittle, polished</i> |
| Hypernymy | including X and other Y | copper google | <i>ones, metals, mines</i> <i>apps, browsers, searches</i> |
| Hyponymy | X like Y | cities browsers | <i>solaris, speyer, medina</i> <i>chrome, firefox, netscape</i> |
| Co-hyponymy | , X , Y , | copper google | <i>malachite, flint, ivory</i> <i>microsoft, bing, yahoo</i> |
| City-State | in X , Y . | portland dallas | <i>oregon, maine, dorset</i> <i>tx, texas, va</i> |
| City-City | from X to Y . | portland dallas | <i>salem, astoria, ogdensburg</i> <i>denton, allatoona, addison</i> |
| Profession | X , a famous Y , | ronaldo monet | <i>footballer, portuguese, player</i> <i>painter, painting, butterfly</i> |

Table 8: Given a context c and a word x , we select the top 3 words y from the entire vocabulary using our scoring function $R(x, y) \cdot C(c)$. The analysis suggests that the model tends to rank correct matches (italics) over others.

pothesis (Harris, 1954) in which the bivariate distribution of target words and contexts is modeled. Our work deviates from the word embedding literature in two major aspects. First, our goal is to represent word *pairs*, not individual words. Second, our new PMI formulation models the *trivariate* word-word-context distribution. Experiments show that our pair embeddings can complement single-word embeddings.

Mining Textual Patterns There is extensive literature on mining textual patterns to predict relations between words (Hearst, 1992; Snow et al., 2005; Turney, 2005; Riedel et al., 2013; Van de Cruys, 2014; Toutanova et al., 2015; Shwartz and Dagan, 2016). These approaches focus mostly on relations between pairs of nouns (perhaps with the exception of VerbOcean (Chklovski and Pantel, 2004)). More recently, they have been expanded to predict relations between unrestricted pairs of words (Jameel et al., 2018; Espinosa Anke and Schockaert, 2018), assuming that each word-pair was observed together during pretraining. Washio and Kato (2018a,b) relax this assumption with a compositional model that can represent any pair, as long as each word appeared (individually) in the corpus.

These methods are evaluated on either intrinsic relation prediction tasks, such as BLESS (Baroni and Lenci, 2011) and CogALex (Santus et al., 2016), or knowledge-base population benchmarks, e.g. FB15 (Bordes et al., 2013). To the best of our knowledge, our work is the first to integrate pattern-based methods into modern high-

performing semantic models and evaluate their impact on complex end-tasks like QA and NLI.

Integrating Knowledge in Complex Models

Ahn et al. (2016) integrate Freebase facts into a language model using a copying mechanism over fact attributes. Yang and Mitchell (2017) modify the LSTM cell to incorporate WordNet and NELL knowledge for event and entity extraction. For cross-sentence inference tasks, Weissenborn et al. (2017), Bauer et al. (2018), and Mihaylov and Frank (2018) dynamically refine word representations by reading assertions from ConceptNet and Wikipedia abstracts. Our approach, on the other hand, relies on a relatively simple extension of existing cross-sentence inference models. Furthermore, we do not need to dynamically retrieve and process knowledge base facts or Wikipedia texts, and just pretrain our pair vectors in advance.

KIM (Chen et al., 2017) integrates word-pair vectors into the ESIM model for NLI in a very similar way to ours. However, KIM’s word-pair vectors contain only hand-engineered word-relation indicators from WordNet, whereas our word-pair vectors are automatically learned from unlabeled text. Our vectors can therefore reflect relation types that do not exist in WordNet (such as *profession*) as well as word pairs that do not have a direct link in WordNet (e.g. *bronze* and *statue*); see Table 8 for additional examples.

7 Conclusion and Future Work

We presented new methods for training and using word *pair* embeddings that implicitly represent background knowledge. Our pair embeddings are computed as a compositional function of the individual word representations, which is learned by maximizing a variant of the PMI with the contexts in which the the two words co-occur. Experiments on cross-sentence inference benchmarks demonstrated that adding these representations to existing models results in sizable improvements for both in-domain and adversarial settings.

Published concurrently with this paper, BERT (Devlin et al., 2018), which uses a masked language model objective, has reported dramatic gains on multiple semantic benchmarks including question-answering, natural language inference, and named entity recognition. Potential avenues for future work include multitasking BERT with `pair2vec` in order to more directly incorporate reasoning about word pair relations into the BERT objective.

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A Implementation Details

Hyperparameters For both word pairs and contexts, we use 300-dimensional word embeddings initialized with FastText (Bojanowski et al., 2017). The context representation uses a single-layer Bi-LSTM with a hidden layer size of 100. We use 2 negative context samples and 3 negative argument samples for each pair-context tuple.

For pre-training, we used stochastic gradient descent with an initial learning rate of 0.01. We reduce the learning rate by a factor of 0.9 if the loss does not decrease for 300K steps. We use a batch size of 600, and train for 12 epochs.⁷

For both end-task models, we use AllenNLP’s implementations (Gardner et al., 2018) with default hyperparameters; we did not change any setting before or after injecting `pair2vec`. We use 0.15 dropout on our pretrained pair embeddings.

B Multivariate Negative Sampling

B.1 Relation to Multivariate PMI

In this appendix, we elaborate on mathematical details of multivariate negative sampling to support our claims in Section 2.2, and also discuss its relation to other PMI multivariate formulations (Table 9).

B.2 Global Objective

Equation (Table 2, J_{3NS}) in Section 2.2 characterizes the local objective for each data instance. To understand the mathematical properties of this objective, we must first describe the global objective in terms of the entire dataset. However, this

⁷On Titan X GPUs, the training takes about a week.

| | | |
|-----------------------|------------------|--|
| (Van de Cruys, 2011) | $SI_1(x, y, c)$ | $\log \frac{P(x, y, \cdot)P(x, \cdot, c)P(\cdot, y, c)}{P(x, \cdot, \cdot)P(\cdot, y, \cdot)P(\cdot, \cdot, c)P(x, y, c)}$ |
| | $SI_2(x, y, c)$ | $\log \frac{P(x, y, c)}{P(x, \cdot, \cdot)P(\cdot, y, \cdot)P(\cdot, \cdot, c)}$ |
| (Jameel et al., 2018) | $SI_3(x, y, c)$ | $\log \frac{P(x, y, c)}{P(x, y, \cdot)P(\cdot, \cdot, c)}$ |
| | $SI_4(x, y, c)$ | $\log \frac{P(x, y, c)P(\cdot, \cdot, c)}{P(x, \cdot, c)P(\cdot, y, c)}$ |
| (This Work) | $MVPMI(x, y, c)$ | $\log \frac{P(x, y, c)}{k_c P(\cdot, \cdot, c)P(x, y, \cdot) + k_x P(x, \cdot, \cdot)P(\cdot, y, c) + k_y P(\cdot, y, \cdot)P(x, \cdot, c)}$ |

Table 9: Multivariate generalizations of PMI.

cannot be done by simply summing the local objective for each (x, y, c) , since each such example may appear multiple times in our dataset. Moreover, due to the nature of negative sampling, the number of times an (x, y, c) triplet appears as a positive example will almost always be different from the number of times it appears as a negative one. Therefore, we must determine the frequency in which each triplet appears in each role.

We first denote the number of times the example (x, y, c) appears in the dataset as $\#(x, y, c)$; this is also the number of times (x, y, c) is used as a positive example. We observe that the expected number of times (x, y, c) is used as a corrupt x example is $k_x P(x, \cdot, \cdot) \#(\cdot, y, c)$, since (x, y, c) can only be created as a corrupt x example by randomly sampling x from an example that already contained y and c . The number of times (x, y, c) is used as a corrupt y or c example can be derived analogously. Therefore, the global objective of our ternary negative sampling approach is:

$$J_{3NS}^{\text{Global}} = \sum_{(x, y, c)} J_{3NS}^{x, y, c} \quad (17)$$

$$J_{3NS}^{x, y, c} = \#(x, y, c) \cdot \log \sigma(S_{x, y, c}) + Z'_{x, y, c} \cdot \log \sigma(-S_{x, y, c}) \quad (18)$$

$$Z'_{x, y, c} = k_x P(x, \cdot, \cdot) \#(\cdot, y, c) J_-^{x, y, c} + k_y P(\cdot, y, \cdot) \#(x, \cdot, c) J_-^{x, y, c} + k_c P(\cdot, \cdot, c) \#(x, y, \cdot) J_-^{x, y, c} \quad (19)$$

$$S_{x, y, c} = R(x, y) \cdot C(c) \quad (20)$$

With the global objective, we can now ask what is the optimal value of $S_{x, y, c}$ (20) by comparing the partial derivative of (17) to zero. This derivative is in fact equal to the partial derivative of (18), since it is the only component of the global objective in which $R(x, y) \cdot C(c)$ appears:

$$0 = \frac{\partial J_{3NS}^{\text{Global}}}{\partial S_{x, y, c}} = \frac{\partial J_{3NS}^{x, y, c}}{\partial S_{x, y, c}}$$

The partial derivative of (18) can be expressed as:

$$0 = \#(x, y, c) \cdot \sigma(-S_{x, y, c}) - Z'_{x, y, c} \cdot \sigma(S_{x, y, c})$$

which can be reformulated as:

$$S_{x, y, c} = \log \frac{\#(x, y, c)}{Z'_{x, y, c}}$$

By expanding the fraction by $1/\#(\cdot, \cdot, \cdot)$ (i.e. dividing by the size of the dataset), we essentially convert all the frequency counts (e.g. $\#(x, y, z)$) to empirical probabilities (e.g. $P(x, y, z)$), and arrive at Equation (1) in Section 2.2.

B.3 Other Multivariate PMI Formulations

Previous work has proposed different multivariate formulations of PMI, shown in Table 9. Van de Cruys (2011) presented specific interaction information (SI_1) and specific correlation (SI_2). In addition to those metrics, Jameel et al. (2018) experimented with SI_3 , which is the bivariate PMI between (x, y) and c , and with SI_4 . Our formulation deviates from previous work, and, to the best of our knowledge, cannot be trivially expressed by one of the existing metrics.