How has belief modality contributed to formal semantics?[*]

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Abstract. Looking back the history of formal treatment of linguistics, we cannot disregard the contribution of possible world semantics. Intensional logic of Montague semantics, DRT (Discourse Representation Theory), mental space, and situation theory are closely related to or compared with the notion of possible world. All these theories have commonly clarified the structure of belief context or uncertain knowledge, employing hypothesized worlds. In this talk, I firstly brief the pedigree of these theories. Next, I will introduce the recent development of modal logic for the representation of (i) knowledge and belief and (ii) time, in which belief modality is precisely discussed together with the accessibility among possible worlds. I will refer to BDI (belief-desire-intention) logic, CTL (computational tree logic), and sphere-based model in belief revision. Finally, I will discuss how these theories could be applied to the further development of analyses of natural language.

1. Introduction

Let me introduce two examples how modality is used in natural language semantics.

Firstly, when non-native English learners first learn two meanings of each of ‘must’ and ‘may’, that is epistemic/deontic readings, they are baffled: what the two meanings have in common?

	must	may
epistemic	have firm knowledge	have vague knowledge
deontic	force obligation	permit tolerantly

The possible world semantics may (or must?) convince some of them, that is,

must	for all possible worlds, denoted by $\Box$
may	some possible worlds exist, denoted by $\Diamond$

Remember in First-Order Logic, $\forall$ (for all) and $\exists$ (some $\cdots$ exist) are interchangeable each other; i. e., $\forall x\ p(x)$ is equal to $\neg\exists\neg p(x)$ (there exists no x such that $\neg p(x)$). In the similar way, ‘$\Box$’ can be replaced by ‘$\neg\Diamond\neg$’ in modal logic. In Japanese, the deontic reading of ‘must’ is translated as

–nakere–ba–naranai.

Here, the first ‘*–nakere–*’ is the negation, and the second ‘*–ba–*’ suggests a case, and the third ‘*–naranai*’ is again the negation. Thus, Japanese represent deontic ‘$\Box$’ by ‘$\neg\Diamond\neg$’.

Secondly, we can introduce a hypothetical belief situation either in the past tense or in the *conjunctive* mood in Indo-European languages, and also we may mention an imaginary current situation in the *conditional*mood,[1] as

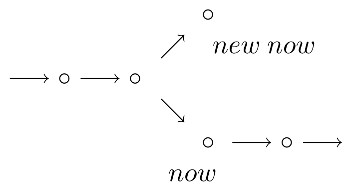
„Wenn ich mehr Zeit hütte, so würde ich Ihnen einen längeren Brief schreiben.“
《S’il venait demain, tu le verrais.》
“Se facesse bel tempo domani, che faresti?”

In English both the conjunctive and the conditional moods have been reduced to the *subjunctive* mood.

“If I were a bird, I would fly to you.”

[1]*Konjunctive* in German.

Why do we employ the past tense when we introduce a counterfactual situation? One explanation is that we consider retreating once to a past (by the conjunctive mood) to annihilate the current situation, and restarting from the past to the different 'now' (by the conditional mood) as in the following figure.



Actually, in English and German which do not possess the conditional mood, 'would' and 'würden' are called *past-future* (future seen from the past). Later, we will explain such a branching time in terms of modal logic.

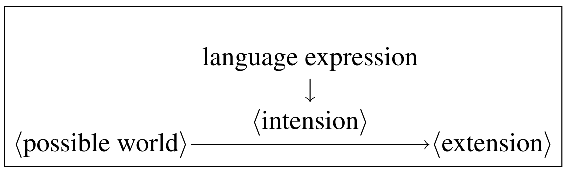
2. Looking back $\cdots$

2.1. Intensional Logic

One of the outstanding achievements of Montague semantics (Dowty, 1979; Dowty, Wall, and Peters, 1981; Gamut, 1991) is the analysis of oblique sentences (or opaqueness), besides proper treatment of quantification, as in

Electra does not know that the man in front of her is her brother. (1)

Electra knows that Orestes is her brother.
The man in front of her is Orestes.
____
Electra knows that the man in front of her is her brother.



de re	= extensional	= reference (Bedeutung)
de dicto	= intensional	= sense (Sinn)

To include intensional logic, we need to revise the syntax of formal language. If α is a language expression, then so is $\hat{}\alpha$. For any w_1 and w_2,

$$[\![\hat{}\alpha]\!]^{w_1} = [\![\hat{}\alpha]\!]^{w_2}$$

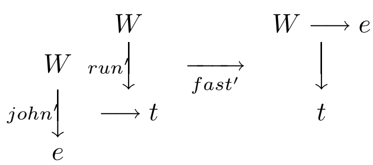
which implies that $[\![\hat{}\alpha]\!]^{w}$ does not depend upon w. s is an index of a possible world, and the meaning of α depends upon w.

$$\alpha\colon a \leftrightarrow \hat{}\alpha\colon \langle s, a\rangle \leftrightarrow \check{}\hat{}\alpha\colon a$$
$$\gamma\{x\} \equiv \check{}\gamma(x) \text{ (Brace convention)}$$

However, do we need to suppose that every lexical item depends on a possible world? In the following lexical items, each s represents an index of a possible world W.

John runs fast. (2)

$$john'\colon \langle s, e\rangle$$
$$run'\colon \langle s, \langle\langle s, e\rangle, t\rangle\rangle$$
$$fast'\colon \langle\langle s, \langle\langle s, e\rangle, t\rangle\rangle, \langle\langle s, e\rangle, t\rangle\rangle$$



Here, the number of s is obviously excessive. In order to avoid needless proliferation of Bennet had suggested the following categories.

(individual)		e
sentence	S	t
common noun	N	$\langle e,t\rangle$
verb phrase / intransitive verb	IV	$\langle e,t\rangle$
transitive verb		
(find, love, eat)	IV/(t/IV)	$\langle\langle s,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle,\langle e,t\rangle\rangle$
transitive verb (believe, assert)	IV/t	$\langle\langle s,t\rangle,\langle e,t\rangle\rangle$
noun phrase / proper noun	t/IV	$\langle\langle s,\langle e,t\rangle\rangle,t\rangle$
determiner	Det	$\langle\langle s,\langle e,t\rangle\rangle,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle$

John believes a penguin flies. (3)

$$\begin{array}{lcl}
\text{John} : t/(t/e) & \Rightarrow & \lambda P[P\{john'\}] \colon \langle\langle s,\langle e,t\rangle\rangle,t\rangle \\
\text{believes} : (t/e)/t & \Rightarrow & believe' \colon \langle\langle s,t\rangle,\langle e,t\rangle\rangle \\
\text{a} : (t/(t/e))/(t/e) & \Rightarrow & \lambda P\lambda Q\exists x[P\{x\}\wedge Q\{x\}] \colon \langle\langle s,\langle e,t\rangle\rangle,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle \\
\text{penguin} : t/e & \Rightarrow & penguin' \colon \langle e,t\rangle \\
\text{flies} : t/e & \Rightarrow & fly' \colon \langle e,t\rangle
\end{array}$$

can be interpreted in the following two ways.

(i) There must be a penguin that flies somewhere.

$$\dfrac{\begin{matrix}\lambda P[P\{john'\}]\\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle\end{matrix} \quad \dfrac{\begin{matrix}believe'\\ \langle\langle s,t\rangle,\langle e,t\rangle\rangle\end{matrix} \quad \dfrac{\dfrac{\begin{matrix}\lambda P\lambda Q\exists x[P\{x\}\wedge Q\{x\}]\\ \langle\langle s,\langle e,t\rangle\rangle,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle\end{matrix} \quad \begin{matrix}penguin'\\ \langle e,t\rangle\end{matrix}}{\begin{matrix}\lambda Q\exists x[penguin'(x)\wedge Q\{x\}]\\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle\end{matrix}} \quad \begin{matrix}fly'\\ \langle e,t\rangle\end{matrix}}{\begin{matrix}\exists x[penguin'(x)\wedge fly'(x)]\\ t\end{matrix}}}{\begin{matrix}believe'(\hat{}\exists x[penguin'(x)\wedge fly'(x)])\\ \langle e,t\rangle\end{matrix}}}{\begin{matrix}believe'(\hat{}\exists x[penguin'(x)\wedge fly'(x)])(john')\\ t\end{matrix}}$$

(ii) The penguin overthere flies.

$$\dfrac{\dfrac{\begin{matrix}\lambda P\lambda Q\exists x[P\{x\}\wedge Q\{x\}]\\ \langle\langle s,\langle e,t\rangle\rangle,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle\end{matrix} \quad \begin{matrix}penguin'\\ \langle e,t\rangle\end{matrix}}{\begin{matrix}\lambda Q\exists x[penguin'(x)\wedge Q\{x\}]\\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle\end{matrix}} \quad \dfrac{\dfrac{\begin{matrix}\lambda P[P\{john'\}]\\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle\end{matrix} \quad \dfrac{\dfrac{\begin{matrix}\lambda P[P\{x_5\}]\\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle\end{matrix} \quad \begin{matrix}fly'\\ \langle e,t\rangle\end{matrix}}{\begin{matrix}fly'(x_5)\\ t\end{matrix}} \quad \begin{matrix}believe'\\ \langle\langle s,t\rangle,\langle e,t\rangle\rangle\end{matrix}}{\begin{matrix}believe'(\hat{}fly'(x_5))\\ \langle e,t\rangle\end{matrix}}}{\begin{matrix}believe'(\hat{}fly'(x_5))(john')\\ t\end{matrix}}}{\begin{matrix}\lambda z[believe'(\hat{}fly'(z))(john')]\\ \langle e,t\rangle\end{matrix}}}{\begin{matrix}\exists x[penguin'(x)\wedge believe'(\hat{}fly'(x))(john')]\\ t\end{matrix}}$$

In the similar way, 'a Democrat' in

Anna believes that a Democrat would win. (4)

can be either within or outside of Anna's belief.

(i) But the man she mentions may not actually be a Democrat.

$$\text{believe}'(\hat{}\exists x[\text{democrat}'(x)\wedge\text{win}'(x)])(\text{anna}')$$

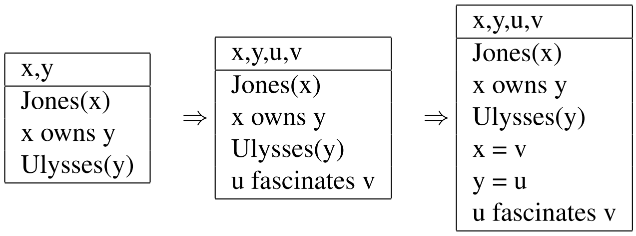
(ii) But she does not know the man's face.

$$\exists x[\text{democrat}'(x)\wedge\text{believe}'(\hat{}\text{win}'(x))(\text{anna}')]$$

2.2. Discourse Representation Theory

Discourse Representation Theory (Kamp and Reyle, 1993) has implemented the partiality and the dynamics of information, i. e., the preceding sentence affects on the succeeding one.

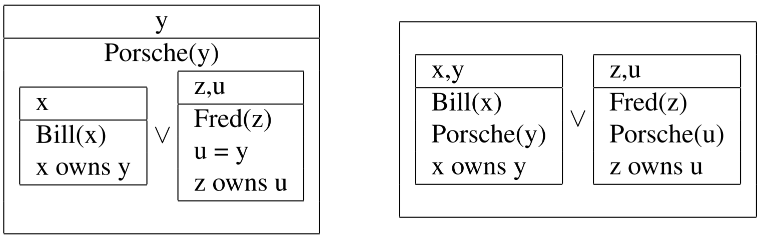
Jones owns Ulysses. It fascinates him. (5)



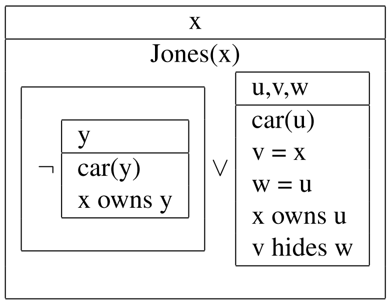
Also, DRT has clarified the process of anaphora resolution.

Bill owns a Porsche or Fred owns it. (6)

- deictic reading: refers what exists in the world.
- anaphoric reading: refers the preceding word, not necessarily an object in the world.



Either Jones doesn't own a car or else he hides it. (7)



A problem about the environment preoccupies every serious politician. (8)

$$\exists y[problem(y) \land \forall x[politician(x) \rightarrow preoccupies(y, x)]]$$
$$\forall x[politician(x) \rightarrow \exists y[problem(y) \land preoccupies(y, x)]]$$

2.3. Mental Space

Principle of ID: we can refer one by the other name if they are mapped by *connector* between mental spaces (Fauconnier, 1994; Fauconnier, 1997). In "Plato is on the top shelf" and "The mushroom omelet left without paying the bill," a book of Plato and the person who ate the mushroom omelet are referred to as metonymy, respectively, in the speaker's mental spaces.

When we consider the distinction of *individual* and *role* (viz. deictic/anaphoric), we develop the theory in the following way.

George thinks the winner will go to Hong Kong. (9)

Introduction of a mental space	'George thinks'
attribute P	'will go to Hong Kong.'
role (r)	the winner

- the attribute of the role itself ··· P(r)
- the attribute of the value of the role ··· P(r(m))

In 1929, the president was a baby. (10)

space introduction	'In 1929'
attribute (P)	'was a baby'
role (r)	the president

(i) In 1929, a certain baby was the president. P(r(M))

(ii) The current president was a baby in 1929. P(F(r(R)))

(iii) In 1929, a president was elected from babies. P(r)

Mental space can be a belief situation, and various beliefs produce multiple interpretations.

Hob thinks$_{M_1}$ a witch has blighted Bob's mare, and Nob believes$_{M_2}$ that she killed Cob's sow. (Geach 1972) (11)

(i) transparent reading: a certain person in R killed the sow and the mare.

(ii) there exists a certain person in M_1, and the correspondent person exists in M_2.

(iii) a person whose role is 'witch' exists in M_1, and a person of the same role exists also in M_2.

Everybody believes that a witch blighted the mare. (Ioup 1977) (12)

- whether *witch* is in 'believe' or not.
- whether is 'every' collective or distributive.
- whether is *witch* a role or a person, while the role is common in every mental space.

(i) A woman exists, and everyone believes that she is a witch. The person who killed the mare is she.

(ii) Everyone believes that there exists a witch, whose image and features are common among them. The witch killed mares.

(iii) Hob believes that a witch is a cute girl with short hair. He believes that the girl killed the mare. Nob believes that a witch is an ugly old woman. She did the crime.

(iv) Everyone believes in witch; but none has seen her, and it is an abstract existence. Everyone just regards that the ominous incident is caused by a witch.

(v) There are two women called Hilda and Brünhilde. Hob believes that Hilda killed the mare, and Nob believes that Brünhilde did it.

3. Belief and Credibility

In this section, we summarize the usage of belief from the viewpoint of artificial intelligence, to consider the agent communication beyond sentential semantics.

3.1. Belief Revision

P. Gärdenfors (Gärdenfors, 1992)

α All European swans are white.

β The bird caught in the trap is a swan.

γ The bird caught in the trap comes from Sweden.

δ Sweden is part of Europe.

From γ and δ, 'the swan in the trap comes from Europe.' With α and β, 'the bird in the trap is white.' What happens if the bird in the trap is black?

Constraints

(i) K, a belief set, should be consistent; i. e., K does not include both of φ and $\neg\varphi$.

(ii) If K logically entails φ ($K \vdash \varphi$), $\varphi \in K$.

$$K = Cn(K) = \{\varphi | K \vdash \varphi\}$$

We regard $K_\perp$ (a set of all the propositions including inconsistency) as a special belief set.
ex. $\{\varphi, \varphi \to \psi\}$ is not a belief set.

(iii) The revision should be minimal. A maximal subset K' that does not entail φ is such that

- $K' \subseteq K$
- $\varphi \notin Cn(K')$
- For any K'' such that $K' \subset K'' \subseteq K$, $\varphi \in Cn(K'')$.

(iv) Sentences with a lower grade of entrenchment should be revised first. Suppose that, by $\varphi \leq \psi$, ψ is more important than, or equal to, φ.
ex. For $\alpha \leq (\alpha \to \beta) \leq (\beta \to \gamma)$,

$$\{\alpha,\ \alpha \to \beta,\ \beta \to \gamma\} * (\neg\gamma) = \{\alpha \to \beta,\ \beta \to \gamma,\ \neg\gamma\}.$$

Revision and Contraction Belief revision $K * \varphi$: in order to maintain consistency, some of the old sentences in K are deleted.

When φ is logically inconsistent,
$K * \varphi = K + \varphi$ ($\equiv K_\perp$) where '$+$' is a simple expansion.
When φ is not inconsistent,
if $\neg\varphi \in K$, $K * \varphi \subset K + \varphi$.
if $\neg\varphi \notin K$, $K * \varphi = K + \varphi$.

Contraction $K \dot{-} \varphi$: some sentences in K is retracted without adding any new facts. In order for the resulting set to be closed under logical consequences some sentences from K must be given up.

3.2. Sphere of Possible Worlds

Suppose that a set of possible worlds $\mathcal{W} = \{w_0, w_2, w_2, \cdots\}$ is partitioned by a credibility function κ; i.e., $\kappa(w_i)$ returns a non-negative integer, and 0 implies that the world is most credible. A subset of possible worlds partitioned by κ is called a *sphere* (Hansson, 1999). Given a proposition φ, we can define the credibility of the proposition by

$$\kappa(\varphi) \equiv \min\{\kappa(w) | w \Vdash \varphi\}.$$

We assume that either $\kappa(\varphi) = 0$ or $\kappa(\neg\varphi) = 0$ for any given φ, and

$$\kappa(\varphi \vee \psi) = \min\{\kappa(\varphi), \kappa(\psi)\}.$$

Suppose K is a belief state, which is modelled by a set of possible worlds $Mod(K)$ where all the propositions in K is true.

$$Mod(K) = \{w | \forall\varphi \in K,\ w \Vdash \varphi\}.$$

If the belief state of a person is K, then (s)he believes that $w \in Mod(K)$ is most credible. Thus,

$$\kappa(w) = 0 \iff w \in Mod(K).$$

In any w, either $\varphi \in w$ or $\neg\varphi \in w$.[2] If (s)he believes ψ, then such w that $w \Vdash \psi$ is in $Mod(K)$. Thus, $\kappa(\neg\psi) > 0$ implies that $Mod(K) \cap Mod(\neg\psi) = \emptyset$ and $Mod(K) \subseteq Mod(\psi)$.

When $w \Vdash \varphi$, let

$$\kappa(w|\varphi) = \kappa(w) - \kappa(\varphi).$$

Because $\kappa(w) \geq \kappa(\varphi)$, $\kappa(w|\varphi) \geq 0$. Let

$$\kappa * (\varphi, \alpha)(w) = \kappa(w|\varphi) \text{ when } w \Vdash \varphi,$$
$$\kappa * (\varphi, \alpha)(w) = \alpha + \kappa(w|\neg\varphi) \text{ when } w \Vdash \neg\varphi.$$

Then, $\kappa * (\varphi, \alpha)(\varphi) = 0$ and $\kappa * (\varphi, \alpha)(\neg\varphi) = \alpha$. α is the firmness of φ; the higher the value, the more firmly φ is believed. When $\alpha > 0$ and $\kappa(\neg\varphi) = 0$, then $\kappa * (\varphi, \alpha)$ is a belief revision.

4. Logic of Belief and Time

In this section, we collect such materials that would contribute to the further development of natural language semantics, regarding belief situation and possible worlds.

4.1. Modal logic

We summarize the fundamentals of modal logic with $\Box$ and $\Diamond$, which represent K (knowledge) and B (belief), and P (past) and F (future).

Axioms If φ is a formula, so is $\Box\varphi$. The following axioms are often employed in modal logic.

K	$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$
$Df\Diamond$	$\Diamond\varphi \leftrightarrow \neg\Box\neg\varphi$
PL	φ, where φ is a tautology.
T	$\Box\varphi \to \varphi$
5	$\Diamond\varphi \to \Box\Diamond\varphi$
4	$\Box\varphi \to \Box\Box\varphi$
B	$\varphi \to \Box\Diamond\varphi$
D	$\Box\varphi \to \Diamond\varphi$

All the above axioms are not independent each other. We can find the following dependency.

$$\{\mathbf{K}, \mathbf{T}\} \vdash \mathbf{D},\ \{\mathbf{K}, \mathbf{T}, \mathbf{5}\} \vdash \mathbf{B},\ \{\mathbf{K}, \mathbf{T}, \mathbf{5}\} \vdash \mathbf{4},\ \{\mathbf{K}, \mathbf{4}, \mathbf{B}\} \vdash \mathbf{5}.$$

Choosing different sets of axioms from the above, we can construct different logic systems, among which there is the following hierarchy of strength.

$$\begin{array}{ccccc} & & KT4 & & \\ & \swarrow & & \nwarrow & \\ K \leftarrow KT & & & & KT5 \\ & \nwarrow & & \swarrow & \\ & & KTB & & \end{array}$$

$$S4 = KT4 = KDT4$$
$$S5 = KT5 = KT45 = KDT5 = KDT45 = KT4B$$

[2] Each world is called to be *maximal*.

Kripke frame Given a set of possible worlds $W = \{w_1, w_2, w_3, \cdots\}$ and the accessibility in them $\mathcal{R} = \{w_1Rw_2, w_1Rw_3, \cdots\}$, $\langle W, \mathcal{R}\rangle$ is called a Kripke frame, which gives a semantics to modal logic.

$$w \models \Box\varphi \iff \forall w'(wRw')\ w' \models \varphi.$$
$$w \models \Diamond\varphi \iff \exists w'(wRw')\ w' \models \varphi.$$

axiom	accessibility	
T	reflective	wRw
5	Euclidean	If wRw' and wRw'', then $w'Rw''$
4	transitive	If wRw' and $w'Rw''$, wRw''
B	symmetric	If wRw' then $w'Rw$
D	serial	$\forall w\exists w'[wRw']$

Ex. Given $\{w_1, w_2, w_3\}$: possible worlds, with:

$$\{w_1Rw_1, w_1Rw_2, w_2Rw_2, w_2Rw_3, w_3Rw_3\}.$$

- $\Box\varphi \to \varphi$ is true.
- $\Box\varphi \to \Box\Box\varphi$ is not always true.

4.2. Knowledge and Belief

Operator K and B The most important notion to model a person's knowledge is the partiality. Suppose that (s)he knows only $\{\varphi, \psi, \chi\}$ are true and are ignorant of others. Because each possible world assigns true/false to every propositions (*maximal*), we can model his/her knowledge state by a collection of possible worlds which assigns 'true' to $\{\varphi, \psi, \chi\}$; other propositions may or may not be true. Thus, the knowledge modality K must be accessible to the all the worlds ($\Box$) which satisfies this condition (Fagin et al., 1995).

$K_i\varphi\ \cdots$ agent i knows φ.
$B_i\varphi\ \cdots$ agent i believes φ.

B_i represents an uncertain knowledge which may not be true in the real world, while K_i satisfies

T $K_i\varphi \to \varphi$.
If i knows φ, φ.

Besides this condition, both of K_i and B_i should satisfy the following axioms.

D $B_i\varphi \to \neg B_i\neg\varphi$.
If i believes φ, (s)he does not believe $\neg\varphi$.
4 $B_i\varphi \to B_iB_i\varphi$.
If i believes φ (s)he believes what (s)he believes.
5 $\neg B_i\varphi \to B_i\neg B_i\varphi$.
If i does not believe φ, (s)he believes that (s)he does not believe φ.

BDI Logic In addition to K_i and B_i, we are arbitrarily add other mental modalities. D_i (desire) and I_i (intention) are often employed, and together with B_i, the logic is called BDI-logic (Bratman, 1987; Cohen, Morgan, and Pollack, 1990). With this, the following expressions are available.

- $B_i\forall x[bordeaux(x) \to mellow(x)]$
  i believes that any Bordeaux wine is mellow.
- $B_iB_j bordeaux(a)$
  i believes that j believes that a is a Bordeaux wine.
- $B_i\forall j[snob(j) \to B_j\forall y[bordeaux(y) \to mellow(y)]]$
  i believes that all the snob guys believes that any Bordeaux wine is mellow.
- $B_i\forall jD_j\neg I_i\ president(i)$
  i believes that everyone desires that i does not intend to be president.

- $B_j \exists x[mellow(x)]$
  j believes that there exists a mellow x (in his/her mind).
- $\exists x[B_j mellow(x)]$
  There exists some x and i believes it is mellow.

4.3. Temporal logic

Priorian temporal logic We had better beginning from Priorian temporal logic (van Benthem, 1991; Goldblatt, 1992). For a formula φ,

- ∘ $P\varphi$ is true. $\equiv$ At some point in the past, φ is true.
- ∘ $F\varphi$ is true. $\equiv$ At some point in the future, φ is true.
- ⋆ $G\varphi \equiv \neg F \neg \varphi$ is true.
- ⋆ $H\varphi \equiv \neg P \neg \varphi$ is true.

Branching Time and CTL As we have mentioned in Introduction, we need not to stick to the linear time. The branching time is far versatile to represent various linguistic information. We employ the following principles.

- Time is discrete.
- Time steps are called states; among which, there is a state called 'now'.
- The past is a straight chaining of states.
- There is the beginning state in the past.
- Time branches to the future.
- The future persists forever.

A sequence of states from the beginning state to a future, following a branch, is called a path. Together with temporal operators, we can define Computational Tree Logic (CTL) (Rao and Georgeff, 1991).

A	For any path,
E	For some path,
$X\varphi$	In the next state, φ.
$\varphi U \psi$	φ until ψ.
$F\varphi$	φ in some future states. ($\equiv \top U \varphi$)
$G\varphi$	φ in all the future states. ($\equiv \neg F \neg \varphi$)

Now the followings are the sentences of CTL.

$$EX\varphi,\ EF\varphi,\ EG\varphi,\ E(\varphi U \psi),\ AX\varphi,\ AF\varphi,\ AG\varphi,\ A(\varphi U \psi).$$

According to the introduction of CTL, the accessibility of belief modality must be redressed as follows. Suppose that branching time is embedded in each possible world. The accessibility relation $\mathcal{B}_i$ is given between possible worlds with regard to a common time, for each agent (Wooldridge, 2000).

$$(w, t)R(w', t) \in \mathcal{B}_i.$$

We simply write $\langle w, t, w' \rangle$ for $(w, t)R(w', t)$.

$$(w, t) \models B_i \varphi \iff \forall w' \langle w, t, w' \rangle \in \mathcal{B}_i,\ (w', t) \models \varphi.$$

5. Toward Communicative Agent

Departing from intra-sentence analysis of belief situation, we can develop a theory of communication, discussing what an agent comes to believe after an informing act of the other agent. First, we introduce the communication channel between agents to implement the notion of uncertainty. In actual mobile agents, the communication eventually might fail by unexpected causes.

The definition of the original *inform* of FIPA (FIPA, 2002) is as follows.

$$\begin{aligned} \text{feasibility pre-condition} &: B_i\varphi \wedge \neg B_i(Bif_j\varphi \vee Uif_j\varphi) \\ \text{rational effect} &: B_j\varphi \end{aligned}$$

A formula $B_j\varphi$ means that agent j believes φ. Also, $Bif_j\varphi$ and $Uif_j\varphi$ are the abbreviations of $B_j\varphi \vee \neg B_j\varphi$ and $U_j\varphi \vee \neg U_j\varphi$, respectively, where $U_j\varphi$ means that agent j is uncertain about φ but the agent supposes that φ is more likely than $\neg\varphi$. However, is the rational effect enough to represent our linguistic communication? There had been two questions:

- The recipient truly comes to know the information if only the sender informs it?
- The recipient should know more besides the information.

We have introduced *channel variables* (Hagiwara, Kobayashi, and Tojo, 2006; de Saeger, Kobayashi, and Tojo, 2007) and added an extra-condition to FIPA's communicability, as follows.

$$\text{feasibility pre-condition (revised)} \quad : \quad B_i\varphi \wedge \neg B_i(Bif_j\varphi \vee Uif_j\varphi) \wedge c_{ij}$$

where c_{ij} is the channel variable between agent i and j. There had been many ways to embed the notion of channel to the logic, e. g., modality, Cartesian product, proposition, and so on. We adopted to employ proposition since we could inform the channel itself as a payload of channel. According to this, in the rational result, agent j should know that i already had known φ, and i would know that j came to know φ. That is,

$$\text{rational effect (revised)} \quad : \quad (B_iB_j\varphi) \vee (B_iB_j\varphi \wedge B_j\varphi \wedge B_jB_i\varphi).$$

This formalization implies that the future states bifurcates into future dependent on whether the communication has been successful or not. This inevitably requires us to incorporate CTL (Section 4.3.) in each possible world. In addition, we need to employ dynamic logic (Wooldridge, 2000) that formalize the state change by inform actions. The knowledge state of each agent in each possible world changes by the inform action, and thus, the accessibility of belief modality also changes, that is the model updating.

Still, there are other possible ways to include the notion of channel. Suppose that the situation s supports the information φ, and φ implies φ' if some extra-condition s' is supplied. Namely,

$$s \circ s' \models \varphi' \iff s \models \varphi \,\&\, \forall s' \sim s, s' \models \varphi \rhd \varphi'$$

where $\circ$ is the connection of situation. This statement can be interpreted in various ways.

- This is a formalization of encryption: if a key s' is supplied, then the received sentence φ is decrypted to a meaningful sentence φ'.

But, in our case,

- If a channel s' is secured in the original environment s, then we can transmit the contents φ' by the form of φ.

Suppose that an agent comes to believe information, based on *history* of communication which is a sequence of inform actions: $H = e_1 \circ e_2 \circ e_3 \cdots$. Then, all the prefixes (initial finite number of sequences) of each history are also regarded as reliable source of knowledge. We call such a collection of prefixes $\mathcal{H}$ *protocol* (Pacuit and Parikh, 2004).

$$(w, H) \models K_i\varphi \iff \forall (w', H') \sim_i (w, H)\ (H' \in \mathcal{H}),\ (w', H') \models \varphi$$

where '$\sim_i$' is the equivalent accessibility relation of agent i. Furthermore, we can define $B_i\varphi$, restricting the accessible worlds by spheres (Section 3.2.). Then, we can avoid the reflectivity **T** (Section 4.1.) to meet the condition of B_i (Section 4.2.).

How can we formalize the inform action of '*witch*' sentences of Geach and Ioup (Section 2.3.), that is, if an agent tells his community that "a witch killed a horse" each member of the community comes to have a different belief state from person to person? The further objective of our study is belief propagation and group revision, where each belief state is elucidated by DRT/mental space.

Finally, notice that we could have rich byproduct in other linguistic phenomena. In the analysis of tense/aspect and event structure, multi-modal logic of precedence and inclusion would reformulate the preceding theory in a more sophisticated way (Tojo, 2006; Koga and Tojo, 2007). Also, if we assume a long history of communication, the protocol may emerge, evolve, and change. Thus, the study may contribute to the study of language evolution (Nakamura, Hashimoto, and Tojo, 2006; Matoba, Nakamura, and Tojo, 2006).

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