# Learning to Correct for QA Reasoning with Black-box LLMs

Jaehyung Kim<sup>1\*</sup> Dongyoung Kim<sup>2</sup> Yiming Yang<sup>3</sup> <sup>1</sup>Yonsei University <sup>2</sup>KAIST <sup>3</sup>Carnegie Mellon University jaehyungk@yonsei.ac.kr

#### Abstract

An open challenge in recent machine learning is about how to improve the reasoning capability of large language models (LLMs) in a blackbox setting, i.e., without access to detailed information such as output token probabilities. Existing approaches either rely on accessibility (which is often unrealistic) or involve significantly increased train- and inference-time costs. This paper addresses those limitations or shortcomings by proposing a novel approach, namely COBB (Correct for improving QA reasoning of Black-Box LLMs). It uses a trained adaptation model to perform a seq2seq mapping from the often-imperfect reasonings of the original black-box LLM to the correct or improved reasonings. Specifically, the adaptation model is initialized with a relatively small open-source LLM and adapted over a collection of sub-sampled training pairs. To select the representative pairs of correct and incorrect reasonings, we formulated the dataset construction as an optimization problem that minimizes the statistical divergence between the sampled subset and the entire collection, and solved it via a genetic algorithm. We then train the adaptation model over the sampled pairs by contrasting the likelihoods of correct and incorrect reasonings. Our experimental results demonstrate that COBB significantly improves reasoning accuracy across various QA benchmarks, compared to the best-performing adaptation baselines.<sup>1</sup>

#### 1 Introduction

Large language models (LLMs) have achieved significant advancements in various NLP tasks, demonstrating exceptional capabilities in understanding and generating text (Anthropic, 2024; OpenAI, 2023; Gemini et al., 2023; Touvron et al., 2023). Nevertheless, LLMs still present notable limitations, such as biased opinions toward specific



Figure 1: **Different black-box LLM adaptation meth-ods.** (a) a model relying on the availability of output token probabilities; (b) a model with increased trainand inference-time costs; (c) COBB (proposed), not requiring output probabilities and is cost-efficient.

groups (Santurkar et al., 2023) or inaccurate predictions for infrequent topics (Kandpal et al., 2023), primarily due to the imperfections in the knowledge acquired during pre-training (Yao et al., 2023). Consequently, it is essential to control and adapt the responses of LLMs to achieve optimal performance for specific use cases. Representative methods include fine-tuning on supervised training datasets (Roziere et al., 2023; Azerbayev et al., 2024) and input-level optimization through prompt engineering and retrieval augmentation (Yang et al., 2024; Kim et al., 2024). However, these approaches require huge training costs or exhibit limited adaptation performance, respectively.

To address these challenges, prior works have focused on training relatively smaller models using responses from LLMs and human supervision, then generating adapted responses while assuming that the LLM parameters are fixed or inaccessible (*i.e.*, black-box). One approach assumes that the output token probabilities are available (Sun et al., 2022; Ormazabal et al., 2023), but this is often unrealistic. Although Sun et al. (2024) recently proposed

<sup>\*</sup>This work is done when Jaehyung was postdoc at CMU. <sup>1</sup>The code will be available at https://github.com/ bbuing9/CoBB.



Figure 2: **An overview of CoBB.** CoBB first collects the multiple reasonings from black-box LLM, and labels them based on the correctness. Among all possible pairs of correct (positive) and incorrect (negative) reasonings, CoBB subsample a few pairs that can maintain the characteristic of the entire set. Then, the adaptation model, initialized with an open-sourced LLM, is trained to increase/decrease the likelihood of positive/negative reasonings.

training a verifier and employing beam search to obtain adapted responses without this assumption, this method results in computationally expensive training and inference pipelines. Alternatively, Ji et al. (2024) introduced a straightforward seq2seq learning framework to enhance the alignment of black-box LLMs. However, extending this framework to other tasks is challenging, particularly in terms of constructing the training dataset and ensuring the effectiveness of the training method.

In this paper, we propose a simple yet efficient framework, learning to **Co**rrect for QA reasoning of **B**lack-**B**ox LLMs (COBB). Our key idea is to learn a seq2seq mapping from the original reasoning of black-box LLM to correct and improved reasoning, by training an adaptation model initialized with a relatively small open-source LLM. After training, the adaptation model can be easily deployed during inference as a single additional module, as illustrated in Figure 1.

Specifically, we firstly sample multiple chainof-thought reasonings from black-box LLM and label their correctness using ground-truth human labels. Then, from all possible pairs of correct and incorrect reasonings, we subsample a few representative pairs that preserve the characteristics of the entire set. To identify such a subset, we formulate an optimization problem that minimizes the statistical divergence between the subset and the entire set, solving it via a genetic algorithm. Finally, using this optimized subset, we train the adaptation model to simultaneously increase the likelihood of correct reasoning and decrease the likelihood of incorrect reasoning for the given input and reasoning. An overview of COBB is presented in Figure 2.

We demonstrate the effectiveness of COBB in improving QA reasoning with black-box LLMs through extensive evaluations on four different QA datasets. For instance, COBB achieved average accuracy improvements of 6.2% and 2.2%, compared to the original black-box gpt-3.5-turbo and previous state-of-the-art adaptation methods, respectively. Furthermore, we found that the adaptation model trained for a specific black-box LLM could generalize to adapt other LLMs, including both API-based and open-source models, which is crucial for efficient deployment in practice. Additionally, our in-depth analyses reveal how COBB improves and corrects the reasoning from the blackbox LLMs. We hope our work provides valuable insights into LLM adaptation research, which is becoming increasingly important for the future success of LLMs in real-world applications.

#### 2 Related Works

#### 2.1 Steering and adapting LLMs' responses

While recent LLMs have demonstrated remarkable success in various tasks, steering and adapting their responses for the specific domain or user is still essential for achieving optimal performance (Santurkar et al., 2023; Salemi et al., 2023; Kandpal et al., 2023). Fine-tuning on human- or machine-labeled datasets is a straightforward approach (Roziere et al., 2023; Azerbayev et al., 2024; Tan et al., 2024), but this method incurs significant costs due to the need to update the vast number of trainable model parameters, particularly for largescale LLMs like GPT-4 (OpenAI, 2023) (>100B parameters). Consequently, prompt engineering (Kojima et al., 2022; Yang et al., 2024) and retrieval augmentation (Kim et al., 2024; Shi et al., 2024) are often preferred, as these methods only require modifying the inputs to LLMs. However, recent observations indicate that input-level modifications alone are insufficient for adequately steering LLMs' responses in the desired direction (Santurkar et al., 2023), likely due to the absence of learnable parameters and learning from human supervision. In this work, we propose an alternative way to steer and adapt LLMs using a trainable model and human supervision, without updating the target LLMs.

#### 2.2 Learning to adapt black-box LLMs

As the scale of LLMs continues to increase, and their parameters often remain inaccessible (i.e., black-box), the need to adapt their responses without updating their parameters has gained significant attention. A common approach involves introducing a relatively small trainable model to learn adaptation from the original responses of the black-box LLM. One line of work focuses on learning to adapt output probabilities (Sun et al., 2022; Ormazabal et al., 2023; Lu et al., 2023; Liu et al., 2024), but this method is impractical when the output probabilities of black-box LLMs are inaccessible. To address this limitation, Sun et al. (2024) propose a verification-based approach, generating the adapted responses in multiple steps via beam search, where scores are calculated using a learned verifier. However, this method increases the costs of training and inference due to the iterative computation between the black-box LLM and the verifier, and deploying the beam search. On the other hand, Ji et al. (2024) demonstrate that a simple seq2seq modeling approach can effectively improve the alignment of black-box LLMs. Despite its effectiveness, this method is limited for the other tasks, in terms of constructing the training dataset and ensuring the effectiveness of the training method. To overcome these limitations, we propose a novel approach to construct an effective training dataset, along with an improved training objective.

# 3 COBB: Learning to Correct for QA Reasoning with Black-box LLMs

In this section, we introduce our framework for learning to **Co**rrect for improving QA reasoning with **B**lack-**B**ox LLMs (COBB). We begin with an overview of the problem setup in Section 3.1. Next, in Section 3.2, we present how to construct an effective dataset for training the adaptation model. This dataset is created by solving an optimization problem using a genetic algorithm, to preserve the characteristics of the entire set of correct and incorrect reasoning pairs from black-box LLM. Finally, we describe a training scheme in Section 3.3, where the adaptation model is trained by contrasting the likelihoods of positive and negative reasonings. The full procedure of COBB is outlined in Algorithm 1, and an overview is provided in Figure 2.

#### 3.1 Preliminaries

Let us denote black-box LLM as  $\mathcal{M}$ , which generates an original output sequence (*e.g.*, reasoning)  $\mathbf{y}_o$  for a given input sequence (*e.g.*, question)  $\mathbf{x}$ , *i.e.*,  $\mathbf{y}_o \sim \mathcal{M}(\cdot | \mathbf{x})$ . Then, our goal is to obtain an adaptation model  $\pi_{\theta}$ , that can generate the adapted output (*e.g.*, improved reasoning)  $\mathbf{y}_a$  from given  $\mathbf{x}$ and  $\mathbf{y}_o$ :

$$\mathbf{y}_a \sim \pi_\theta(\cdot | \mathbf{x}, \mathbf{y}_o). \tag{1}$$

For example, Ji et al. (2024) initialize  $\pi_{\theta}$  with a pre-trained open-sourced LLM, and fine-tune it by minimizing a supervised cross-entropy:

$$\mathcal{L}_{\text{SFT}}(\theta) = -\log \pi_{\theta}(\mathbf{y}_a | \mathbf{y}_o, \mathbf{x}), \qquad (2)$$

where  $\mathbf{x}, \mathbf{y}_o, \mathbf{y}_a \sim \mathcal{D} = \{(\mathbf{x}^i, \mathbf{y}^i_o, \mathbf{y}^i_a)\}_{i=1}^N$ . To improve the LLM's alignment regarding helpfulness and harmlessness, Ji et al. (2024) construct  $\mathcal{D}$  using weaker LLMs (*e.g.*, Alpaca-7B) for  $\mathbf{y}_o$  and stronger LLMs (*e.g.*, GPT-4) or human annotations for  $\mathbf{y}_a$ , respectively. In our case, we assume that a human-annotated QA dataset  $\mathcal{Q} = \{(\mathbf{q}^i, \mathbf{a}^i)\}_{i=1}^{N_q}$  is available, where  $\mathbf{q}$  is question of target task and  $\mathbf{a}$  is the ground-truth answer. Then, our goal is to train adaptation model  $\pi_{\theta}$ , which is also initialized with open-sourced LLM, using  $\mathcal{Q}$  and obtain the improved reasoning with  $\mathcal{M}$  for this task.

# **3.2** Optimizing dataset to learn from effective reasoning pairs via genetic algorithm

Collecting and labeling of training pairs. To train adaptation model  $\pi_{\theta}$  using Q, we first collect positive and negative reasonings from  $\mathcal{M}$ . Specifically, for each  $\mathbf{q}, \mathbf{a} \sim Q$ , we sample K different reasonings  $\{\mathbf{y}_{\text{cot},k}\}_{k=1}^{K}$  using few-shot chain-of-thought prompt  $\mathbf{p}_{\text{cot}}$  (Wei et al., 2022):

$$\mathbf{y}_{\text{cot},k} \sim \mathcal{M}(\cdot | \mathbf{q}, \mathbf{p}_{\text{cot}}).$$
 (3)

Then, if the prediction by  $y_{cot}$  is *correct* (*i.e.*, equal to the answer a), we assign this reasoning to the

positive reasoning set,  $\mathcal{Y}_{pos}$ . If not, we assign this reasoning to the negative reasoning set,  $\mathcal{Y}_{neg}$ . Remarkably, we denote that there are some cases where (1)  $\mathcal{M}$  can't generate any correct reasoning (*i.e.*,  $\mathcal{Y}_{pos} = \emptyset$ ) or (2) there is no incorrect reasoning (*i.e.*,  $\mathcal{Y}_{neg} = \emptyset$ ). For (1), we utilize answeraugmented prompting (Zelikman et al., 2022) to generate the reasoning to support the given answer a and augment  $\mathcal{Y}_{pos}$  with it. For (2), we randomly select the reasoning of another sample and augment  $\mathcal{Y}_{neg}$  with it, to fully utilize the samples in  $\mathcal{Q}$ .

Solving optimization to find effective reasoning pairs via genetic algorithm. With the collected  $\mathcal{Y}_{pos}$  and  $\mathcal{Y}_{neg}$ , we want to construct the training dataset  $\mathcal{D}$  to train  $\pi_{\theta}$ , composed of the triplet of the question **q**, positive reasoning  $\mathbf{y}_p$ , and negative reasoning  $\mathbf{y}_n$ . However, the number of possible combinations between positive and negative reasonings is quadratically increased, *i.e.*,  $|\mathcal{Y}_{pos}| \times |\mathcal{Y}_{neg}|$ ; it can be too large trained within the limited iterations and there can be large redundancy within the constructed dataset. To tackle this challenge, we propose to subsample a few representative positive and negative reasoning pairs, that can preserve the characteristics of all the possible combinations.

Specifically, for each **q**, let denote the set of all the possible pairs of positive and negative reasonings as  $\mathcal{P} = \mathcal{Y}_{pos} \times \mathcal{Y}_{neg}$ . Then, for each pair in  $\mathcal{P}$ , we calculate its likelihood difference under  $\pi_{\theta}$ :

$$\mathbf{P} = \{ \pi_{\theta}(\mathbf{y}_p | \mathbf{q}) - \pi_{\theta}(\mathbf{y}_n | \mathbf{q}) \mid \mathbf{y}_p, \mathbf{y}_n \in \mathcal{P} \}.$$
(4)

Then, we propose to find a subset  $\mathcal{P}_{sub} \subset \mathcal{P}$  which minimizes  $d(P_{sub}, P)$ , where  $P_{sub}$  is obtained from  $\mathcal{P}_{sub}$  similar to Eq. 4 and  $d(\cdot, \cdot)$  is a distance between two sets. Here, we assume the elements of both P and  $P_{sub}$  are samples from two different normal distributions and then consider 2-Wasserstein distance (Givens and Shortt, 1984) between them:

$$d(\mathbf{P}_{\mathtt{sub}}, \mathbf{P}) = (\mu - \mu_{\mathtt{sub}})^2 + (\sigma - \sigma_{\mathtt{sub}})^2, \quad (5)$$

where  $\mu, \sigma^2$  are the empirical mean and variance of P and  $\mu_{sub}, \sigma_{sub}^2$  the empirical mean and variance of P<sub>sub</sub>, respectively. We empirically observe that this 2-Wasserstein distance is better than other possible metrics such as KL divergence.

However, finding  $P_{sub}$  that minimizes the distance (Eq. 5) is non-trivial, as this selection of representative samples problem is NP-hard (Gamez et al., 2008), and the current objective includes the quadratic terms. To mitigate these challenges, we

#### Algorithm 1 COBB algorithm

**Input:** Black-box LLM  $\mathcal{M}$ , adaptation model  $\pi_{\theta}$ , target QA dataset  $\mathcal{Q} = \{\mathbf{q}^i, \mathbf{a}^i\}_{i=1}^{N_q}$ , number of sampling K, genetic algorithm iterations T, training iteration  $T_{\text{train}}$ , learning rate  $\eta$ 

/\* Construction of dataset  $\mathcal{D}$  \*/  $\mathcal{D} = \emptyset$ for  $\mathbf{q}, \mathbf{a} \in \mathcal{Q}$  do /\* Collect reasonings \*/  $\mathcal{Y} = \{\mathbf{y}_{\texttt{cot},\texttt{k}}\}_{k=1}^{K}, \ \mathbf{y}_{\texttt{cot},\texttt{k}} \sim \mathcal{M}(\cdot|\mathbf{q}, \textbf{p}_{\texttt{cot}})$ /\* Get reasoning pairs \*/  $\mathcal{P} = \mathcal{Y}_{\mathtt{pos}} imes \mathcal{Y}_{\mathtt{neg}} \leftarrow \mathcal{Y}$  with a  $\mathbf{P} \leftarrow \mathcal{P}$  with  $\pi_{\theta}$  (Eq. 4) /\* Subset selection \*/  $\mathcal{P}_{sub} \leftarrow \text{Genetic}(\mathcal{P}, \mathbf{P}, |\mathcal{Y}_{neg}|, T) \text{ (Eq. 6)}$  $\mathcal{D} \leftarrow \mathcal{D} \cup (\{\mathbf{q}\} \times \mathcal{P}_{\mathsf{sub}})$ end for /\* Train adaptation model  $\pi_{\theta}$  \*/ for t = 0 to  $T_{\texttt{train}} - 1$  do /\* Loss for mini-batch \*/  $\mathcal{L}_{\texttt{train}}(\theta, \mathcal{B}) \leftarrow \mathcal{B} \sim \mathcal{D}, \pi_{\theta} \text{ (Eq. 7)}$ /\* Update adaptation model \*/  $\theta \leftarrow \theta - \eta \nabla_{\theta} \mathcal{L}_{\text{train}}$ end for **return** trained adaptation model  $\pi_{\theta}$ 

use a genetic algorithm (Holland, 1992), which progressively optimizes the solution by iterating (1) acquiring a new candidate by perturbing the current solution and (2) updating the solution when the candidate achieves a better optimization objective. We consider a new random sampling of  $\mathcal{P}_{sub}$  as the perturbation, and obtain  $\mathcal{P}_{sub}^*$  after T iterations:

$$\mathcal{P}_{\text{sub}}^* = \text{Genetic}(\mathcal{P}, \mathbf{P}, N_s, T), \qquad (6)$$

where  $N_s$  is the size of the subset and a detailed description is presented in Algorithm 2.<sup>2</sup> We observed that the genetic algorithm found a good solution within a few iterations and it only requires small additional computations (see Table 11). With  $\mathcal{P}_{sub}^*$ , we construct the dataset  $\mathcal{D} = \{(\mathbf{q}^i, \mathbf{y}_n^i, \mathbf{y}_p^j)\}_{i=1}^N$ , where  $(\mathbf{y}_n, \mathbf{y}_p) \in \mathcal{P}_{sub}^*$  for  $\mathbf{q}$ .<sup>3</sup>

# **3.3** Learning to correct by contrasting likelihoods of reasoning pairs

With the constructed dataset  $\mathcal{D}$ , we train the adaptation model  $\pi_{\theta}$  to learn the seq2seq mapping from

<sup>&</sup>lt;sup>2</sup>During the experiments, we fix  $N_s$  as  $|\mathcal{Y}_{neg}|$ .

<sup>&</sup>lt;sup>3</sup>We remark that there can be duplicated  $\mathbf{q}^{i}$ , as multiple reasoning pairs are constructed for each  $\mathbf{q}$ ,  $\mathbf{a}$  from  $\mathcal{Q}$ .

the original reasoning from black-box LLM  $\mathcal{M}$ to the correct and improved reasoning. While the supervised training with a cross-entropy (Eq. 2) is considerable (Ji et al., 2024), we observed that this approach could be limited, especially when the target task requires careful discrimination between positive and negative reasonings. Therefore, we propose to further use the negative reasoning  $y_n$ to lower its likelihood in the output space of  $\pi_{\theta}$ , while simultaneously increasing the likelihood of the positive reasoning  $y_p$ . Specifically, we construct our training objective  $\mathcal{L}_{\texttt{train}}$  using Odds Ratio Preference Optimization (ORPO) (Hong et al., 2024), which enables single-stage learning from pair-wise preference data, without the reference models. Namely, we treat  $y_p$  as preferred output and  $\mathbf{y}_n$  as dispreferred output:

$$\mathcal{L}_{\texttt{train}}(\theta, \mathcal{D}) = \mathbb{E}_{\mathcal{D}}[\mathcal{L}_{\texttt{SFT}}(\theta) + \lambda \cdot \mathcal{L}_{\texttt{OR}}(\theta)], \quad (7)$$

$$\mathcal{L}_{\text{DR}}(\theta) = -\log \sigma \left( \log \frac{\text{odds}_{\theta}(\mathbf{y}_p | \mathbf{x})}{\text{odds}_{\theta}(\mathbf{y}_n | \mathbf{x})} \right), \quad (8)$$

where  $\sigma$  is a sigmoid function,  $\lambda$  is a hyperparameter, and  $\text{odds}_{\theta}(\mathbf{y}|\mathbf{x}) = \frac{\pi_{\theta}(\mathbf{y}|\mathbf{x})}{1-\pi_{\theta}(\mathbf{y}|\mathbf{x})}$ . Here, we use the concatenation of question  $\mathbf{q}$  and reasoning  $\mathbf{y}$  (for both  $\mathbf{y}_p$  and  $\mathbf{y}_n$ ) as the input  $\mathbf{x}$ , to model the seq2seq mapping between the original reasoning from  $\mathcal{M}$  (input) and the refined reasoning through  $\pi_{\theta}$  (output), conditioned on  $\mathbf{q}$ . As shown in Figure 3, incorporating  $\mathbf{y}_n$  via Eq. 7 effectively suppresses the increasing likelihood of negative reasonings.

#### 4 **Experiments**

#### 4.1 Setups

Datasets and metrics. Following the recent work (Sun et al., 2024), we evaluate COBB on four different question-answering (QA) tasks, requiring adaptation on mathematical (GSM8K), implicitreasoning (StrategyQA), truthful (TruthfulQA), and scientific (ScienceQA) domains. We use the train and test splits by Sun et al. (2024). To generate the reasonings for each dataset, we follow the previous chain-of-thought prompts used in prior work (Sun et al., 2024), except GSM8K. In the case of GSM8K, we adopt a complex prompt (Fu et al., 2023), as it yields higher accuracy compared to the previous one. During the evaluation, we sample K = 5 chain-of-thought reasoning for each test question, and measure (1) the average accuracy (Avg.) across 5 reasonings, and (2) the accuracy of prediction from majority voting among them

(Maj@5). For TruthfulQA, we report the average of the accuracies on helpfulness and informativeness (True + Info) following (Sun et al., 2024), along with the majority voted accuracy. More details of the datasets are in Appendix A.1.

Baselines. We compare COBB against several extensive baselines as follows: (1) Target Blackbox LLM: without adaptation, we use the reasoning from the target black-box LLM  $\mathcal{M}$ , (2) Initial Open-sourced LLM: we generate the reasoning from the open-sourced LLM, which is used to initialize the adaptation model  $\pi_{\theta}$ , (3) Supervised Fine-Tuning (SFT):  $\pi_{\theta}$  is fine-tuned with a given QA dataset Q, (4) Chain-of-Thought Distillation (CoT Distill) (Li et al., 2023a): instead of answer a in original Q, the positive reasoning  $y_p$  is used as the output label for input q to fine-tune  $\pi_{\theta}$ . (5) Aligner (Ji et al., 2024):  $\pi_{\theta}$  is fine-tuned to learn a seq2seq mapping from the concatenation of q and  $\mathbf{y}_n$  to  $\mathbf{y}_p$  via cross-entropy loss (Eq. 2), (6) BBox-Adapter (Sun et al., 2024): learning a verifier model to deploy beam search and generate the adapted reasoning in iterative inference and verification steps.

Implementation details. For the target black-box LLM  $\mathcal{M}$ , we mainly consider gpt-3.5-turbo-0125, and it is used to generate the reasoning for the training adaptation model. To initialize the adaptation model  $\pi_{\theta}$ , we consider Mistral-7B-inst-v2 (Jiang et al., 2023). For BBox-Adapter (Sun et al., 2024), we follow the original experimental setups in the official codes. For other adaptation methods including COBB, we commonly fine-tune  $\pi_{\theta}$  for 5 epochs with a batch size of 128, using an Adam optimizer (Kingma and Ba, 2015) with a learning rate of  $1 \times 10^{-5}$  and cosine scheduler with a warm ratio of 0.03. Also, we use a temperature of 1.0to sample the reasoning for each question. For the hyper-parameters of COBB, we used fixed values of  $\lambda = 0.1, T = 1000, K = 10$ . Here, we generate half of the reasonings from  $\mathcal{M}$ , and the remaining half from the initial  $\pi_{\theta}$  for efficiency.

#### 4.2 Main results

Table 1 summarizes the experimental results on four different QA tasks, by adapting the reasoning of gpt-3.5-turbo (*i.e.*, target blackbox LLM  $\mathcal{M}$ ). First, it is observed that Mistral-7b-inst-v2, which is used to initialize the adaptation model  $\pi_{\theta}$ , originally exhibits significantly lower performance than the target

Table 1: **Main results.** Test performance with different adaptation methods across four different QA tasks. Here, gpt-3.5-turbo is a target black-box LLM and Mistral-7b-inst-v2 is used to initialize the adaptation model. The best and second best scores are highlighted in **bold** and <u>underline</u>, respectively.

Dataset $(\rightarrow)$	Strat	egyQA	GS	M8K	Truthfu	lQA	Scien	iceQA
Methods ( $\downarrow$ ) / Metrics ( $\rightarrow$ )	Avg.	Maj@5	Avg.	Maj@5	True + Info	Maj@5	Avg.	Maj@5
Target Black-box LLM	70.92	71.62	76.25	79.23	71.40	73.00	81.24	83.00
Initial Open-sourced LLM	61.40	62.88	43.50	51.40	75.40	78.50	65.52	66.60
SFT	66.11	66.38	49.67	55.19	61.40	65.00	85.52	85.80
CoT Distill	67.14	70.31	58.01	65.20	80.00	83.00	77.08	80.40
Aligner	58.69	59.83	76.42	79.83	79.60	<u>83.50</u>	74.92	81.00
BBox-Adapter	71.27	73.36	78.79	83.09	73.80	73.00	81.96	83.60
CoBB (Ours)	74.93	75.11	<u>78.59</u>	85.14	82.90	85.50	88.00	89.20

Table 2: **Transferability with CoBB.** Test performance of original and adapted reasonings of different LLMs. Here, we use the adaptation model which is initialized with Mistral-7b-inst-v2 and trained to adapt gpt-3.5-turbo, and it is denoted by \*. The best scores for each LLM are highlighted in **bold**.

Dataset $(\rightarrow)$	StrategyQA		GS	M8K	TruthfulQA		ScienceQA	
Methods ( $\downarrow$ ) / Metrics ( $\rightarrow$ )	Avg.	Maj@5	Avg.	Maj@5	True + Info	Maj@5	Avg.	Maj@5
Claude-3-Haiku	72.05	72.93	<b>83.85</b>	83.17	67.90	68.00	82.00	82.40
Claude-3-Haiku <b>+ CoBB</b> *	<b>72.58</b>	<b>76.42</b>	81.73	<b>86.66</b>	<b>81.20</b>	<b>83.50</b>	<b>87.40</b>	<b>88.80</b>
Mistral-7B-inst-v2	61.40	62.88	43.50	51.40	75.40	78.50	65.52	66.60
Mistral-7B-inst-v2 + CoBB*	<b>70.31</b>	<b>74.24</b>	<b>56.03</b>	<b>66.57</b>	<b>84.10</b>	<b>86.00</b>	<b>85.56</b>	<b>87.70</b>
Phi-3-mini-4k-inst	62.27	62.88	<b>82.18</b>	86.05	78.10	79.00	83.32	85.80
Phi-3-mini-4k-inst + CoBB*	<b>70.22</b>	<b>74.67</b>	78.10	<b>86.13</b>	<b>84.00</b>	<b>88.00</b>	<b>86.84</b>	<b>88.80</b>
Gemma-1.1-7B-it	57.12	55.90	49.54	53.22	60.90	61.50	71.84	74.60
Gemma-1.1-7B-it + CoBB*	<b>72.66</b>	<b>73.36</b>	<b>61.85</b>	<b>69.14</b>	<b>82.00</b>	<b>84.50</b>	<b>87.12</b>	<b>88.40</b>

black-box LLM. However, the model's performance is largely increased after the adaptation to the target task, regardless of the methods; it shows the importance of an additional adaptation stage for black-box LLM, using both the ground-truth human supervision and the collected reasonings of the black-box LLM. In addition, among these adaptation methods, one can observe that COBB yields the largest improvements in most cases. Specifically, COBB exhibits 6.2%/7.0% average accuracy (Acc.) and the majority voted accuracy (Maj@5) improvements for the target black-box LLM, on average across 4 QA tasks. Furthermore, compared to the strongest baselines, COBB exhibits 2.2%/2.3% average improvements, respectively.

Remarkably, as shown in Table 3, COBB requires much smaller costs during the training of the adaptation model ( $\approx 20\%$ ) and the test-time inference ( $\approx 7\%$ ), compared to the previous state-of-the-art method (*BBox-Adapter*).<sup>4</sup> This is be-

cause CoBB directly learns a seq2seq modeling while BBox-Adapter learns to verify through the sampling and beam search. These results indicate that CoBB could serve as a more powerful yet cost-efficient adaptation method.

We further demonstrate the advantage of COBB regarding the transferability to various LLMs; namely, we deploy the adaptation model, trained with gpt-3.5-turbo (in Table 1), to adapt reasonings of other LLMs including other API-based black-box LLM (Claude-3-Haiku (Anthropic, 2024)) and open-source LLMs (Mistral-7B-inst-v2 (Jiang et al., 2023), Phi-3-mini-4k-inst (Abdin et al., 2024), Gemma-1.1-7B-it (Team et al., 2024)).<sup>5</sup> This result is presented in Table 2. Here, one can observe that COBB successfully adapts the reasoning of various LLMs and improves the accuracies overall, even without the specific adaptation to

 $<sup>^4</sup>We$  follow the official implementation and hyper-parameters by the authors in <code>https://github.com/</code>

haotiansun14/BBox-Adapter.

<sup>&</sup>lt;sup>5</sup>For open-source LLMs, we only use the generated reasoning without access to the internal model weights or output probabilities to treat them as black-box LLM.



(c) Effect of different coefficient  $\lambda$ (a) Without contrastive training objective (b) With contrastive training objective

Figure 3: Effect of contrasting likelihoods. Change of the likelihood of  $\pi_{\theta}$  for positive and negative reasonings in the training dataset (a) without / (b) with the contrastive training objective (Eq. 8) on ScienceQA. (c) Test accuracy of the adapted reasonings of gpt-3.5-turbo on ScienceQA with varied coefficient  $\lambda$ .

Table 3: Cost efficiency of COBB. Costs consumed for API calls (for target black-box LLM) during training and evaluation of BBox-Adapter and COBB. For the evaluation, we sample 5 reasoning per sample. Costs are calculated with the official per-token price.

Table 4: Unseen task generalization. Test performance
on MATH-sub. We use the adaptation model which is
initialized with Mistral-7b-inst-v2 and trained
to adapt gpt-3.5-turbo on GSM8K, and it is de-
noted by $^{\dagger}$ . The best results are highlighted in <b>bold</b> .

	Strate	gyQA	Scienc	eQA
Methods	Train (\$)	Eval (\$)	Train (\$)	Eval (\$)
BBox-Adapter	5.92	1.87	5.98	3.90
CoBB	1.25	0.14	1.01	0.25

the target LLM. To be specific, COBB exhibits 9.1%/11.1% average accuracy (Acc.), and the majority voted accuracy (Maj@5) improvements, on average across 4 LLMs and 4 QA tasks. On the other hand, it is observed that the average accuracy on GSM8K is slightly decreased when the target LLM already exhibits a stronger performance than the LLM used to generate the training data. From this result and the overall improvements with the transferred adaptation model, it is inferred that the knowledge included in the constructed training dataset is more important for the effectiveness of the adaptation model, rather than the specific type of LLM used to construct the data. We present the results with the standard deviation in Appendix B.

#### Additional analyses with COBB 4.3

In this section, we provide additional analyses of CoBB. We conduct the experiments on StrategyQA and ScienceQA, by setting gpt-3.5-turbo as the target black-box LLM  $\mathcal{M}$  and Mistral-7b-inst-v2 as the initialization model for the adaptation model  $\pi_{\theta}$  in default. Ablation study. To validate the effectiveness of the proposed components of COBB in Section 3, we perform the ablation experiments by decomposing our framework with two components of (1) the dataset construction via genetic algorithm (Eq. 6)

Methods	Avg.	Maj@5
Target Black-box LLM	59.60	65.33
$\mathrm{CoBB}^\dagger$	60.80	70.67

and (2) the training objective to contrast the likelihood of positive and negative reasonings (Eq. 8). We denote these components as Gen. and Con., respectively. For comparison, we consider random subsampling when the genetic algorithm selection is not applied. Additionally, we set  $\lambda = 0$  when the contrastive training objective is not used.

The results are presented in Table 5. Here, it can be observed that using the contrastive training objective significantly improves the accuracy of the adapted reasoning, and the improvements are further enhanced when the adaptation model is trained on more representative reasoning pairs. At the same time, it is observed that the proposed dataset construction is not effective without the contrastive training objective. These results indicate that adjusting the likelihood of  $\pi_{\theta}$  is crucial to successfully learning the adaptation, and effective dataset construction aids by guiding where to adjust. We further present Figure 3 to reveal the effect of contrastive training objective. Here, one can observe that the likelihood of negative reasoning is even increased compared to the initial stage, when the cross-entropy loss is only used with the positive reasoning (Eq. 2). However, by incorporating the contrastive objective, this problem is clearly resolved. One can also observe that its effectiveness is not sensitive to the choice of  $\lambda$ .

Table 5: Ablation study. Average test accuracy of adapted reasoning of gpt-3.5-turbo with different configurations of the proposed components in COBB.

Methods	Con.	Gen.	StrategyQA	ScienceQA
Aligner	X	X	58.69	74.92
	×	1	59.30	73.36
	✓	×	71.35	86.60
CoBB	<ul> <li>Image: A second s</li></ul>	<ul> <li>Image: A second s</li></ul>	74.93	88.00

Table 6: **Different initialization for adaptation model.** Average test accuracy with different open-source LLMs for the initialization of the adaptation model  $\pi_{\theta}$ . StQA is StrategyQA and ScQA is ScienceQA, respectively.

Initialization	CoBB	StQA	ScQA
No adaptation	N/A	70.92	81.24
LLaMA-2-7B-chat-hf	×	56.24	46.48
	✓	63.13	76.84
Mistral-7B-inst-v2	×	61.40	65.52
	✓	74.93	88.00

**Generalization of COBB to unseen task.** We conduct additional experiments to verify whether the trained adaptation model via COBB can be generalized to an unseen yet relevant task. To be specific, we apply the adaptation model of COBB, which is trained on GSM8K, to another mathematical QA, MATH-sub (Hendrycks et al., 2021; Hosseini et al., 2024), at the test time. As MATH-sub consists of unseen diverse subfields of mathematics, we use it to evaluate the generalization to unseen tasks. The results are presented in Table 4, we found COBB successfully improves the accuracy of reasoning from the black-box LLM in this unseen task.

**Effect of different initialization for**  $\pi_{\theta}$ **.** Next, we conduct experiments to reveal the importance of the choice of open-sourced LLM to To this end, we use LLaMA2 initialize  $\pi_{\theta}$ . (LLaMA-2-7B-chat-hf), which has a similar number of trainable parameters as the originally used Mistral (Mistral-7B-inst-v2), for the initialization and measure the average accuracy before/after applying COBB. The results are presented in Table 6; when COBB is applied ( $\checkmark$ ), it indicates that  $\pi_{\theta}$  is trained with each initialization LLM and used to adapt the reasoning from gpt-3.5-turbo. One can first notice that the accuracy of LLaMA2 is largely worse than Mistral. While the accuracy of the adapted reasoning with LLaMA2 is significantly increased, it still fails to improve the accuracy of the reasonings from the

Table 7: Additional baselines. Test performance with different adaptation methods on ScienceQA. Here, gpt-3.5-turbo is a target black-box LLM and Mistral-7b-inst-v2 is used to initialize the adaptation model. The best scores are highlighted in **bold**.

Methods	Avg.	Maj@5
SFT (full, pos)	76.48	78.80
SFT (full, both)	68.28	71.80
CoBB (full)	85.64	86.40
CoBB	88.00	89.20

black-box LLM, unlike Mistral. This result implies that pre-trained knowledge within the open-source LLM is crucial to learning the correction of QA reasoning via COBB, and we could benefit from the continued advances of open-source LLMs.

**Comparison with more baselines.** In addition, we conduct additional experiments on ScienceQA, to compare CoBB with another line of baselines that train the adaptation model with a full set of constructed contrastive pairs. Specifically, we consider the following three different training methods to construct these baselines: (1) supervised fine-tuning (SFT) that uses a cross-entropy loss with only positive reasoning in all contrastive pairs (*SFT (full, pos)*), (2) SFT with both positive and negative reasoning in all contrastive pairs (*SFT (full, pos)*), and (3) contrastive objective with both positive and negative reasoning in the contrastive pairs (*CoBB (full)*). The results are presented in Table 7.

Here, it's observed that the original COBB significantly outperforms these new baselines. Also, using the selected reasoning pairs with the proposed dataset construction shows better performance than using full data. This improvement might be related to the risk of overfitting, as using the full set of contrastive pairs yields more training iterations with each reasoning. For the empirical evidence for this, we observed that the training log odds (log  $\frac{odds_{\theta}(y_P|x)}{odds_{\theta}(y_n|x)}$  in Eq. 8) is saturated in the early epochs when training on a full set (COBB (full)) compared to the subset (COBB).

**Inference-time efficiency of COBB.** We further investigate the efficiency of COBB in terms of time consumption, compared to the previous state-ofthe-art method (BBox-Adapter (Sun et al., 2024)) to adapt black-box LLM. We remark that the cost comparison in terms of money spent from API calls is presented in Table 3. Since COBB and BBox-Adapter have different training datasets, we only **Question:** Which is a sentence fragment? **Choices:** ["Our car was stuck in traffic we were late.", "A strong, bold girl."]

**Originally incorrect reasoning from gpt-3.5-turbo:** Our car was stuck in traffic we were late. - This is a sentence fragment because it does not have a subject and a complete predicate. A strong, bold girl. - This is a complete sentence with a subject (girl) and a predicate (strong, bold). #### 0

**Corrected reasoning by COBB**: The first option is a run-on sentence because it contains two independent clauses without proper punctuation. "Our car was stuck in traffic" and "we were late" are both independent clauses that need to be separated. The second option, "A strong, bold girl," is a sentence fragment since it lacks a main subject or verb to make it a complete sentence. Therefore, the answer is: The sentence fragment is "A strong, bold girl." #### 1

Figure 4: **Qualitative example on ScienceQA.** Example of the question, original reasoning from black-box LLM (gpt-3.5-turbo), and the adapted reasoning by CoBB. More examples are presented in Appendix C.

Table 8: Inference time consumption. Consumed time (seconds, lower is better) for the inference of all examples in StrategyQA. Here, gpt-3.5-turbo is a target black-box LLM and Mistral-7b-inst-v2 is used to initialize the adaptation model.

	Source of Time Consumption						
Methods	API Calls	GPU-level Inference	Overall				
BBox-Adapter	6728	116	6844				
COBB (Original)	840	2932	3772				
CoBB (vLLM)	840	215	1055				

Table 9: **In-depth analyses of COBB.** Analyses to deeper understand of how COBB works. We compare the changes in several metrics between the reasonings from gpt-3.5-turbo (original) and the reasonings by COBB (adapted) on ScienceQA.

Metrics	gpt-3.5-turbo	CoBB
Acc. of orig. correct $(\uparrow)$	100.0	92.22
Acc. of orig. incorrect $(\uparrow)$	0.000	69.72
$\pi_{\theta}(\mathbf{y})$ of orig. correct ( $\uparrow$ )	0.767	0.756
$\pi_{\theta}(\mathbf{y})$ of orig. incorrect ( $\uparrow$ )	0.264	0.685
Cosine similarity (↓)	0.910	0.906
Self-BLEU (↓)	0.472	0.503

compare the time consumption for the inference on the same test dataset of StrategyQA, using the same A6000 GPU. The results are summarized in Table 8. Here, the adaption with our method only requires 55% of inference time. This reduction is from the simplicity of the proposed framework, while BBox-Adapter requires the iterations of API calls and verifier model inference. In addition, we would like to emphasize that the inference time with our method could be significantly reduced, by incorporating the existing techniques for efficient GPU-level inference. For example, when changing the inference code from the original Huggingface's generation function (Huggingface, 2019) to vLLM (Kwon et al., 2023), one can reduce 92.7% of GPUlevel inference time without the loss of accuracy.

**In-depth analyses of COBB.** Lastly, we conduct additional analyses to deeply understand how COBB works. Specifically, we try to answer the following question: how COBB changes the (1) correctness, (2) likelihood, and (3) diversity of the reasonings of the black-box LLM. The corresponding experimental results are presented in the top, middle, and bottom rows of Table 9, respectively. First, it is observed that COBB mostly keeps the correctness of the originally correct reasonings ( $100 \rightarrow 92.2$ ), while significantly improving the incorrect ones ( $0 \rightarrow 69.72$ ). Also, such behavior is observed in terms of the likelihood; when we

measure the likelihood of reasoning y with the trained adaptation model  $\pi_{\theta}(\mathbf{y})$ , one can observe that the likelihood of originally correct reasonings is maintained and incorrect reasonings' is largely increased. Then, one potential concern might be that COBB loses the diversity within the original reasonings, and generates the identical adapted reasonings. But, as shown in Table 9, it is observed that the diversity of original reasonings is well-preserved after the adaptation via COBB; it demonstrates that COBB can understand the context within the original reasoning and properly incorporate it during the adaptation.

### 5 Conclusion

In this paper, we proposed COBB, a simple yet effective framework for learning to correct QA reasoning of black-box LLM. We propose to learn a seq2seq mapping from the original reasoning of black-box LLM to correct and improved reasoning, by training a relatively small adaptation model with the newly proposed dataset construction and training objective. Our experiments demonstrate the effectiveness of COBB across various QA tasks and LLMs. Therefore, we believe our framework can contribute to various real-world applications that require the adaptation of black-box LLMs.

# Limitations

While CoBB shows promising results in our experiments, several limitations must be acknowledged. First, the effectiveness of COBB heavily depends on the quality of the training pairs and the capability of the initial open-source LLM. While the proposed dataset construction via genetic algorithm aims to select representative pairs, the initial set of collected reasonings might still be biased (Santurkar et al., 2023) or insufficiently diverse (Wang et al., 2023) depending on the black-box LLM used for the reasoning generation, potentially affecting the adaptation model's performance. Moreover, the effectiveness of our framework largely depends on the specific open-source LLM used to initialize the adaptation model, as shown in Table 6. While this reliance may be seen as a limitation, it also highlights a strength of our framework, as it can benefit from the rapid advancements in open-source LLM development in recent days.

Secondly, COBB requires ground-truth human labels to judge the correctness of reasonings, which can be resource-intensive and time-consuming to obtain, especially for large-scale datasets. Additionally, while CoBB demonstrates transferability across different LLMs, the adaptation performance may vary based on the specific characteristics and pre-training knowledge of the target LLMs. Lastly, the computational efficiency of COBB, although improved compared to the baselines, can still pose challenges as it yields the fine-tuned open-source LLMs per each task which has a large number of model parameters. To address this issue, incorporating the parameter-efficient fine-tuning techniques (Dettmers et al., 2023) or distillation into a smaller model (Gu et al., 2024) could be effective.

## **Broader Impact and Ethical Implications**

We strongly believe that COBB framework has the potential to provide significant positive impacts across various real-world applications. For instance, depending on the user, the interested domain could be varied, such as education, healthcare, and finance (Gan et al., 2023; Clusmann et al., 2023; Li et al., 2023b). However, as highlighted in the recent study (Kandpal et al., 2023), the accuracy of LLMs could be not sufficient if the considered domain is less frequently trained. In such a case, our framework offers an efficient solution for generating domain-specific responses without incurring huge costs, compared to the conventional solution

## of continual training (Singhal et al., 2023a,b).

At the same time, however, there are also some potential negative impacts. A primary concern is the risk of reinforcing existing biases present in the training data, whether they originate from the target black-box LLM, the human-annotated datasets, or the pre-trained knowledge of the open-source LLM used for initialization. For example, recent research has shown that state-of-the-art LLMs even exhibit biases towards specific groups (Santurkar et al., 2023). If this kind of undesired bias is not properly removed during the training of the adaptation model, then our framework could reproduce or amplify the bias. We believe that this problem could be mitigated by incorporating additional filtering stages during the dataset construction, training, or inference (Le Bras et al., 2020; Dong et al., 2023; Zhang and Zhou, 2024), and we remain this problem for the future direction.

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#### A Additional Experimental Details

This section provides more details about the experimental setups in Section 4. We note that all of our experiments are conducted with 2 NVIDIA RTX A6000 GPUs (48GB memory) and AMD EPYC 7313 16-core Processor (3.7 max CPU Ghz).

#### A.1 Datasets

Here, we present more details of four QA tasks used in our experiments. The overall dataset description and statistics are presented in Table A.1. Also, the examples of this dataset are presented in Figure 5. We follow the same train and test splits of the previous work (Sun et al., 2024).

- StrategyQA (Geva et al., 2021) is a binary true/false (T/F) QA benchmark that emphasizes implicit multi-hop reasoning for strategy-based questions. Here, a strategy indicates the skill to derive subquestions from the main question. Notably, the questions in StrategyQA are not constrained to specific decomposition patterns and include strategies employed by humans in answering questions. Therefore, this benchmark requires models to infer unspoken premises and perform multiple reasoning steps to produce accurate answers, especially in cases where the answers are not immediately clear from the given information.
- GSM8K (Cobbe et al., 2021) is a collection of high-quality, linguistically diverse grade school math word problems. Each problem requires between 2 and 8 steps to solve and involves a series of calculations using basic arithmetic operations to determine the final answer. Consequently, solving these problems necessitates multi-step reasoning and mathematical computations based on the problem's context.
- TruthfulQA (Lin et al., 2022) is a dataset to assess a model's ability to produce truthful, factual, and accurate answers. It targets the common issue of AI models generating plausible yet incorrect responses, challenging their ability to recognize and maintain truthfulness. For evaluation, we follow the prior work (Sun et al., 2024) that utilizes prompting.
- ScienceQA (Lu et al., 2022) multi-modal question-answering dataset centered on science topics, consists of annotated answers, lectures, and explanations. The dataset originally included around 21,000 multi-modal multiplechoice questions. In our experiments, we ad-

here to the setup by Sun et al. (2024), which excludes questions needing image input and randomly selects 2,000 questions for training and 500 for testing, each sourced from the dataset's original train and test subsets, respectively.

#### A.2 Baselines

In this section, we provide more details about each baseline. First, to generate the chain-of-thought reasoning (Wei et al., 2022) from LLMs for both the test and the construction of the training dataset of COBB, we adopt the previously used few-shot chain-of-thought prompt (Sun et al., 2024; Fu et al., 2023). The used prompts are presented in Figure 6 In addition, as noticed in Section 4.1, we sample 5 chain-of-thought reasonings per each test sample. To this end, we use sampling with a temperature for the following baselines: Target Black-box LLM, Initial Adaptation Model, SFT, CoT Distill, and BBox-Adapter. Here, we commonly use a temperature of 1.0 except *BBox-Adapter*, as we use the optimized hyper-parameter (including temperature) by the authors for this baseline.<sup>6</sup> In the case of Aligner and CoBB (Ours), we generate the adapted reasoning with a greedy decoding (*i.e.*, temperature of 0.0), as both methods receive the generated reasoning by black-box LLMs as the input and hence already includes sufficient diversity. In addition, for both methods, we consider the likelihood-based filtering mechanism for GSM8K dataset, where the adapted reasoning is only accepted when its likelihood is higher than the original one. Also, we commonly evaluate the performance of each method after the training (i.e., last checkpoint).

#### A.3 COBB

First, in Algorithm 2, we present the full procedure of how the genetic algorithm is used to construct the dataset, which is introduced in Section 3.2. In addition, regarding the choice of hyper-parameters, we use  $\lambda = 0.1$  as this value was most efficient for the alignment fine-tuning in the original ORPO paper (Hong et al., 2024); also, with the experiments on ScienceQA (shown in Figure 3(c)), we similarly confirmed that this value is mostly effective. For the iterations of genetic algorithm T, we use T = 1000 as it sufficiently decreases the target objective (Eq. 5) within considerable time. For example, the dataset construction for ScienceQA with T = 1000 consumes 72 seconds and one can

<sup>&</sup>lt;sup>6</sup>https://github.com/haotiansun14/ BBox-Adapter

Table 10: **Dataset desciption and statistics.** More descriptions and statistics of four QA datasets used in experiments.

Dataset	Domain	Answer type	# of Train Data	# of Test Data
StrategyQA	Implicit-reasoning	Binary (T/F)	2,061	229
GSM8K	Mathematics	Open-ended text	7,473	1,319
TruthfulQA	Truthfulness	Open-ended text	717	100
ScienceQA	Science	Multiple-choice	2,000	500

#### StrategyQA

Question: Karachi was a part of Alexander the Great's success? Answer: True

GSM8K

Question: Natalia sold clips to 48 of her friends in April, and then she sold half as many clips in May. How many clips did Natalia sell altogether in April and May?

<u>Answer</u>: Natalia sold 48/2 = (48/2) = 24 + 24 = 100 clips in May. Natalia sold 48 + 24 = (48 + 24) = 72 + 24 = 100 may. #### 72

#### TruthfulQA

Question: Who is the richest person in the world who didn't finish high school? Best Answer: I have no comment.

#### ScienceQA

Question: Which figure of speech is used in this text? Dr. Shelton is unhappy with her new assistant because simple tasks, like fetching coffee, take him years to finish. Choices: [anaphora, hyperbole]

Answer: hyperbole

Figure 5: Examples of datasets. Examples from four QA datasets used in experiments.

Table 11: **Trajectory of objective during optimization.** Change of the target objective value (Eq. 5) with the varied number of iteration T. We measure this on StrategyQA dataset and average across 10 different runs.

	T = 1	T = 10	T = 100	T = 1000
<b>Objective</b> (Eq. 5)	0.0161	0.0073	0.0066	0.0065

confirm that the improvement from more iterations is almost saturated (see Table 11).

# **B** Additional Quantitative Results

In this section, we present more quantitative results that are not presented in the main draft.

#### **B.1** Results with standard deviation

First, we present the standard deviation for the results in Tables 1 and 2. Specifically, we additionally calculate the standard deviation of the accuracies among five different reasonings; hence, it is only calculated for the average accuracy (Acc.), not for the majority voted accuracy (Maj@5). These results are presented in Tables 12 and 13. Here, one can observe that the improvement by COBB is clear without the overlap between confidence inter-

#### Algorithm 2 Genetic algorithm

**Input:** Set of positive and negative reasoning pairs  $\mathcal{P}$ , likelihood difference between positive and negative reasonings (in  $\mathcal{P}$ ) P, number of the selected samples in the subset  $N_s$ , number of iterations T

/\* Set initial dummy value \*/
dmin = 10000
for t = 0 to T - 1 do
 /\* Sampling new subset \*/
 I<sup>t</sup> = Randint(0, |\mathcal{P}|, (N\_s, ))
 
$$\widetilde{\mathcal{P}}^t \leftarrow \mathcal{P}[I^t]$$
,  $\widetilde{\mathbf{P}}^t \leftarrow \mathbf{P}[I^t]$ 
 /\* Calculating objective \*/
 d<sup>t</sup>  $\leftarrow d(\widetilde{\mathbf{P}}^t, \mathbf{P})$  from Eq. 5
 /\* Update the selection \*/
 if d<sup>t</sup> < d\_{min} then
  $\mathcal{P}_{sub} \leftarrow \widetilde{\mathcal{P}}^t$ , d\_min  $\leftarrow d^t$ 
 end if
end for
return  $\mathcal{P}_{sub}$ 

vals in most cases.

Table 12: Main results with standard deviation. Test performance with different adaptation methods across four different QA tasks. gpt-3.5-turbo is a target black-box LLM and Mistral-7b-inst-v2 is used to initialize the adaptation model. The best and second best scores are highlighted in **bold** and <u>underline</u>, respectively.

Dataset $(\rightarrow)$	StrategyQA	GSM8K	TruthfulQA		ScienceQA	
Methods ( $\downarrow$ ) / Metrics ( $\rightarrow$ )	Avg.	Avg.	True	Info	Avg.	
Target Black-box LLM Initial Open-sourced LLM	$\begin{array}{c} 70.92{\pm}1.22 \\ 61.40{\pm}0.59 \end{array}$	$76.25{\pm}0.59 \\ 43.50{\pm}0.21$	$\begin{array}{c} 62.60{\pm}1.60\\ 66.40{\pm}2.58\end{array}$	$\begin{array}{c} 80.20{\pm}1.74\\ 84.40{\pm}0.75\end{array}$	$81.24{\pm}0.65$ $65.52{\pm}0.39$	
SFT CoT Distill Aligner BBox-Adapter COBB (Ours)	$\begin{array}{c} 66.11 {\pm} 0.59 \\ 67.14 {\pm} 1.13 \\ 58.69 {\pm} 1.96 \\ \underline{71.27} {\pm} 1.76 \\ 74.93 {\pm} 3.26 \end{array}$	$\begin{array}{c} 49.67{\pm}1.09\\ 58.01{\pm}1.35\\ 76.42{\pm}0.39\\ \textbf{78.79}{\pm}0.83\\ 78.59{\pm}0.85\end{array}$	$67.60\pm2.33$ $70.00\pm2.58$ $69.80\pm1.41$ <b>72.20</b> ±1.33 <b>72.20</b> ±1.41	55.20±2.58 90.00±2.64 89.40±1.17 75.40±2.06 93.60±1.17	85.52±0.16 77.08±2.34 74.92±1.32 81.96±0.92 88.00±0.61	

Table 13: **Transferability of COBB with standard deviation.** Test performance of adapted reasoning of different LLMs. Here, we use the adaptation model which is initialized with Mistral-7b-inst-v2 and trained to adapt gpt-3.5-turbo, which is indicated with \*. The best scores for each LLM are highlighted in **bold**.

Dataset $(\rightarrow)$	StrategyQA	GSM8K	Truth	fulQA	ScienceQA
Methods ( $\downarrow ) \ / \ Metrics ( \rightarrow )$	Avg.	Avg.	True	Info	Avg.
Claude-3-Haiku Claude-3-Haiku <b>+ CoBB</b> *	$\begin{array}{c} 72.05{\pm}1.70\\ \textbf{72.58}{\pm}1.80\end{array}$	<b>83.85</b> ±0.72 81.73±0.43	67.00±1.41 <b>71.60</b> ±2.94	68.80±3.87 90.80±1.17	82.00±0.40 87.40±0.67
Mistral-7B-inst-v2 Mistral-7B-inst-v2 + CoBB*	61.40±0.59 <b>70.31</b> ±2.80	$\begin{array}{c} 43.50{\pm}0.21\\ \textbf{56.03}{\pm}0.82\end{array}$	66.40±2.58 <b>75.00</b> ±1.79	84.40±0.75 93.20±1.47	65.52±0.39 85.56±0.50
Phi-3-mini-4k-inst Phi-3-mini-4k-inst + COBB*	$\begin{array}{c} 62.27{\pm}1.48 \\ \textbf{70.22}{\pm}2.28 \end{array}$	<b>82.18</b> ±0.79 78.10±1.15	63.60±2.33 <b>75.00</b> ±2.28	$92.60{\pm}1.02\\93.00{\pm}2.19$	$\begin{array}{c} 83.32{\pm}0.45\\ \textbf{86.84}{\pm}0.66\end{array}$
Gemma-1.1-7B-it Gemma-1.1-7B-it + CoBB*	57.12±0.97 <b>72.66</b> ±1.58	$\begin{array}{c} 49.54{\pm}0.92\\ \textbf{61.85}{\pm}0.83\end{array}$	63.80±2.14 73.00±2.10	$58.00{\pm}2.10\\ \textbf{91.00}{\pm}1.67$	$71.84{\pm}1.04\\ \textbf{87.12}{\pm}0.41$

## **B.2** GPT-4 with CoBB

Next, we verify the potential of COBB to improve the state-of-the-art black-box LLM. To this end, we consider qpt-40 (OpenAI, 2024) as a target black-box LLM and generated the adapted reasoning using (1) the adaptation model trained with gpt-3.5-turbo (in Table 1) and (2) the newly trained adaptation model with gpt-40. The results are presented in Table 15. First, it is observed that the adapted reasonings by COBB exhibit better performance compared to the ones from CoBB\*. These results show the importance of using better source LLM in constructing the dataset, as it can contribute to providing extensive and deeper knowledge. Nevertheless, even using gpt-40 for dataset construction, the performance improvement is quite limited under the current choice of COBB. We suspect that this limitation might stem from the limited capacity of the current adaptation model,

which is initialized by Mistral-7b-inst-v2, as implicitly evidenced in Table 6. Therefore, if stronger open-source LLM, in terms of the number of model parameters and the overall performance, could be used as the adaptation model, we believe that our framework can learn the adaptation, even for the state-of-the-art black-box LLMs.

#### **B.3** Effect of augmenting negatives

As described in Section 3.2, if there is no incorrect reasoning for the given question, we use the reasoning for the other question as artificial negative reasoning and augment  $\mathcal{Y}_{neg}$  with it. To verify the effect of such augmentation, we conduct additional experiments by removing such augmentation. Specifically, we only utilize the correct reasoning for the questions with no incorrect reasoning, using the cross-entropy loss (contrastive loss for the others). The results are presented in Table

Table 14: Effect of augmentation of negative examples. Test performance COBB and its variant on ScienceQA. Here, gpt-3.5-turbo is a target black-box LLM and Mistral-7b-inst-v2 is used to initialize the adaptation model. The best scores are highlighted in **bold**.

Methods	Avg.	Maj@5
CoBB	88.00	89.20
COBB (no aug)	87.40	88.20

Table 15: **GPT-4 with COBB.** Test performance of original and adapted reasoning of gpt-40 (OpenAI, 2024) under COBB. COBB\* uses the adaptation model trained for gpt-3.5-turbo (Table 1) and COBB trains new adaptation model by generating the dataset with gpt-40 and use it for the adaptation. The best scores are highlighted in **bold**.

	StrategyQA		ScienceQA	
Methods	Avg.	Maj@5	Avg.	Maj@5
GPT-40	80.09	80.79	92.08	91.20
+ CoBB*	75.55	78.60	88.00	88.40
+ CoBB	75.63	79.04	92.16	93.00

14. Here, one can observe that the proposed sampling and augmenting  $Y_{neg}$  consistently improves the performance. We conjecture that this improvement might be caused by stabilizing the training, as this augmentation could work as an easy data augmentation (Swayamdipta et al., 2020). For the evidence, when measuring the variance of log odds  $(\log \frac{odds_{\theta}(y_p|x)}{odds_{\theta}(y_n|x)})$  in Eq. 8) during the training, we observe that the training with the proposed augmentation exhibits a significantly lower variance of log odds than the training without the augmentation (17.37 v.s. 27.30).

#### **B.4** More analyses on subsampling method

Here, we present the results to demonstrate the effectiveness of the proposed objective in Eq. 7 and the constructed subset by optimizing this. Specifically, we would like to answer the below questions: *Q1. How representative of the subsamples compared to the full set?* To show the representativeness of the subsamples, we first measure the 2-Wasserstein distance between the subset and the entire set (Eq. 7), which captures the statistical divergence of the contrastive reasoning pairs within these two sets. Consequently, the lower distance under this metric would indicate better representa-

Table 16: **Representativeness of subset.** Some measurements to evaluate the representativeness of the constructed subset on ScienceQA.

Methods	2-Wasserstein Distance $(\downarrow)$	kNN Distance ( $\downarrow$ )	Rouge-L (†)
Random	0.0270	0.550	0.507
CoBB	0.0113	0.544	0.514

Table 17: **Representativeness of subset.** Some measurements to evaluate the representativeness of the constructed subset on ScienceQA.

Methods	2-Wasserstein Distance $(\downarrow)$	kNN Distance ( $\downarrow$ )	Rouge-L (†)
kNN distance (opt)	0.0238	0.503	0.544
Rouge-L (opt)	0.0323	0.569	0.593
CoBB	0.0113	0.544	0.514

tiveness of the subsamples. We compare subsets with our method and the random subsampling on ScienceQA, and the results are presented in the 2nd column of Table 16. Here, our method shows a significantly lower distance than the random subset, by successfully selecting the subsets that optimize this metric.

We also used two additional metrics to comprehensively evaluate the representativeness of the subset to the entire set: kNN distance (DeVries et al., 2020) and Rouge-L (Wang et al., 2023). Specifically, for all reasonings of each question in the entire set, we calculate the 12 distance to the nearest neighbor (k = 1, except itself) in the constructed subset, on the sentence embedding space (Wang et al., 2024). Similarly, we calculate Rouge-L to the nearest neighbor to incorporate another similarity metric. For these metrics, the nearest neighbor is found within the reasonings with the same type (positive or negative). As shown in the 3rd and 4th columns of Table 16, the subset constructed with our method also is more representative than the random subset, under these metrics. These results confirm that our method successfully constructs the subset that is representative of the entire set.

# *Q2. How the optimization goals and methodologies affect the representativeness of the subsamples?*

To compare the effect of the different optimization goals on the representativeness of the subsamples, we construct new subsets that directly optimize the above metrics, kNN distance and Rouge-L. Table 17 shows the evaluated results with these subsets. Here, it is observed that the evaluated values highly depend on the optimization goals, and there is no single method that outperforms others in all metrics. Therefore, to further compare the effectiveness of each optimization goal, we additionally

Table 18: Effect of objective for subset selection. Test performance CoBB and its variant on ScienceQA. Here, gpt-3.5-turbo is a target black-box LLM and Mistral-7b-inst-v2 is used to initialize the adaptation model. The best scores are highlighted in **bold**.

Methods	Avg.	Maj@5
CoBB (kNN Dist.)	86.44	88.00
CoBB (Rouge-L)	85.84	87.80
CoBB	88.00	89.20

Table 19: **In-depth analyses of COBB.** Analyses to deeper understand of how COBB works. We compare the changes in several metrics between the reasonings from gpt-3.5-turbo (original) and the reasonings by COBB (adapted) on Strategy dataset.

Metrics	gpt-3.5-turbo	CoBB
Acc. of orig. correct $(\uparrow)$	100.0	87.44
Acc. of orig. incorrect $(\uparrow)$	0.000	44.44
$\pi_{\theta}(\mathbf{y})$ of orig. correct ( $\uparrow$ )	0.776	0.777
$\pi_{\theta}(\mathbf{y})$ of orig. incorrect ( $\uparrow$ )	0.696	0.757
Cosine similarity (↓)	0.926	0.920
Self-BLEU (↓)	0.521	0.490

train the adaptation model using newly constructed subsets (which optimize kNN distance and Rouge-L) and compare their test accuracies of the adapted reasonings of gpt-3.5-turbo on ScienceQA. As shown in Table 18, the original dataset that minimizes 2-Wasserstein distance consistently yields higher accuracy, and it further validates our design of the optimization goal.

## **B.5** In-depth analyses on more datasets

Lastly, we further present the in-depth analysis results on StrategyQA in Table 19, similar to Table 9 which is conducted on ScienceQA. Here, similar results are observed and it indicates that the interpretation presented in Section 4.3 continuously makes sense across the different tasks.

# **C** Additional Qualitative Examples

In this section, we present the additional qualitative examples of how the original reasoning from gpt-3.5-turbo is adapted and corrected in Figures 7, 8, and 9. From these examples, one can notice that COBB successfully corrects the reasoning while preserving lexical diversity and grammatical correctness.

#### <StrategyQA>

Use the step-by-step method as shown in the examples to answer the question. Break down the problem into smaller parts and then provide the final answer (Yes/No) after "####".

#### Example 1:

Q: Karachi was a part of Alexander the Great's success?

A: Karachi is a city in modern day Pakistan. Krokola was an ancient port located in what is now Karachi. Alexander the Great stationed his fleet in Krokola on his way to Babylon. Alexander the Great defeated Darius and conquered Babylon before expanding his empire. #### Yes.

Example 2:

Q: Was P. G. Wodehouse's favorite book The Hunger Games?

A: P. G. Wodehouse died in 1975. The Hunger Games was published in 2008. #### No.

Your Question: Q: <QUESTION> A:

#### <GSM8K>

Question: Angelo and Melanie want to plan how many hours over the next week they should study together for their test next week. They have 2 chapters of their textbook to study and 4 worksheets to memorize. They figure out that they should dedicate 3 hours to each chapter of their textbook and 1.5 hours for each worksheet. If they plan to study no more than 4 hours each day, how many days should they plan to study total over the next week if they take a 10-minute break every hour, include 3 10-minute snack breaks each day, and 30 minutes for lunch each day?

Let's think step by step. Angelo and Melanie think they should dedicate 3 hours to each of the 2 chapters, 3 hours x 2 chapters = 6 hours total. For the worksheets they plan to dedicate 1.5 hours for each worksheet, 1.5 hours x 4 worksheets = 6 hours total. Angelo and Melanie need to start with planning 12 hours to study, at 4 hours a day, 12/4 = 3 days. However, they need to include time for breaks and lunch. Every hour they want to include a 10-minute break, so 12 total hours x 10 minutes = 120 extra minutes for breaks. They also want to include 3 10-minute snack breaks, 3 x 10 minutes = 30 minutes. And they want to include 30 minutes for lunch each day, so 120 minutes for breaks + 30 minutes for snack breaks + 30 minutes for lunch = 180 minutes, or 180 / 60 minutes per hour = 3 extra hours. So Angelo and Melanie want to plan 12 hours to study 4 days to allow for all the time they need. The answer is 4. ... (7 more examples) ...

Question: <QUESTION> Let's think step by step.

#### <TruthfulQA>

You are a helpful, respectful and honest assistant. Always answer as helpfully as possible, while being safe. Your answers should not include any harmful, unethical, racist, sexist, toxic, dangerous, or illegal content. Please ensure that your responses are socially unbiased and positive in nature. If a question does not make any sense, or is not factually coherent, explain why instead of answering something not correct. If you don't know the answer to a question, please don't share false information.

Q: <QEUSTION>

#### A:

#### <ScienceQA>

Use the step-by-step method as shown in the example to answer the question. Respond to the question by adhering to the given format: provide step-by-step reasoning (one sentence per line), and then give the final answer after "####".

Example:

Question: Which figure of speech is used in this text? Dr. Shelton is unhappy with her new assistant because simple tasks, like fetching coffee, take him years to finish.

Choices:

0: anaphora

1: hyperbole

Answer: The text uses hyperbole, an obvious exaggeration that is not meant to be taken literally. Take him years to finish is an exaggeration, since it probably does not take him entire years to fetch coffee. #### 1

Your Question: <QEUSTION>

Figure 6: **Chain-of-thought prompt.** Chain-of-thought prompt (Wei et al., 2022) to extract the reasoning of LLM. Except for the case of GSM8K, we use the exact same one from the prior work (Sun et al., 2024). In the case of GSM8K, we adopt the complex prompt (Fu et al., 2023), as it yields higher accuracy compared to the previous one. Here, we only present a single few-shot example for GSM8K. Full 8 examples are presented in (Fu et al., 2023).

Question: Does this passage describe the weather or the climate? Where Rita lives, winds blowing from the northeast are rare in July.

Choices: ["weather", "climate"]

**Originally incorrect reasoning from gpt-3.5-turbo:** The passage mentions a specific instance about winds blowing from the northeast in July. This occurrence relates to a short-term atmospheric condition, not a long-term pattern. Hence, the passage describes weather, as it talks about a specific condition at a given time. #### 0

**Corrected reasoning by COBB**: The passage describes the climate. Climate refers to the typical weather conditions of a region over a long period, while weather refers to the conditions at a specific time. In this passage, Ritaś location being described as where "winds blowing from the northeast are rare in July" indicates a recurring pattern of wind direction and strength over a longer period known as climate. #### 1

**Question:** According to Newton's third law, what other force must be happening? **Choices:** ["The paper clip is pulling on the fridge magnet."] The paper clip is pushing on the fridge magnet."]

**Originally incorrect reasoning from gpt-3.5-turbo:** According to Newton's third law, for every action, there is an equal and opposite reaction. If the paper clip is pushing on the fridge magnet, then the fridge magnet must also be pushing back on the paper clip with an equal force. Therefore, the force happening is: The paper clip is pushing on the fridge magnet. #### 1

**Corrected reasoning by COBB**: The force of the fridge magnet is attracting the paper clip. According to Newton's third law, for every action, there is an equal and opposite reaction. Since the paper clip is being attracted to the fridge magnet, the fridge magnet must also be attracting the paper clip with an equal force. #### 0

**Question:** Which word would you find on a dictionary page with the following guide words? cove - cylinder **Choices:** ["credit", "cattle"]

**Originally incorrect reasoning from gpt-3.5-turbo:** The word "cattle" would appear between "cove" and "cylinder" since "cattle" comes after "cove" alphabetically but before "cylinder." #### 1

**Corrected reasoning by CoBB**: The word "credits" would be found at the bottom of the dictionary page, not on the actual page with the guide words. The guide words indicate that the word must come after "cove" and before "cylinder" alphabetically. "Credit" falls between "cove" and "cylinder" alphabetically, so it would be found on the dictionary page. ##### 0

Figure 7: More qualitative examples on ScienceQA. Examples of the question, original reasoning from black-box LLM (gpt-3.5-turbo), and the adapted reasoning by COBB.

Question: Does Olympia Washington share name with Hephaestus's workshop location?

**Originally incorrect reasoning from gpt-3.5-turbo:** Olympia is the capital city of the state of Washington in the United States. The workshop of Hephaestus, the Greek god of blacksmiths and craftsmen, was located on Mount Olympus in Greek mythology. #### No.

**Corrected reasoning by COBB**: Olympia is the capital city of the state of Washington in the United States. Hephaestus's workshop was located on Mount Olympus in Greek mythology. Both Olympia and Mount Olympus are associated with the Greek gods and mythology. Olympia, Washington, and Mount Olympus share names with Hephaestus's workshop location. #### Yes.

Question: Would Library of Alexandria need less shelf space than Library of Congress?

**Originally incorrect reasoning from gpt-3.5-turbo:** The Library of Alexandria was founded in the 3rd century BCE and was one of the largest and most significant libraries of the ancient world. The Library of Congress was established in 1800 and is the largest library in the world by shelf space and number of books. #### No.

**Corrected reasoning by CoBB**: Library of Alexandria was one of the largest libraries in the ancient world, while the Library of Congress is the national library of the United States and is much larger than the Library of Alexandria. The Library of Alexandria was estimated to have held around 40,000 scrolls, while the Library of Congress has over 3 million books and numerous electronic resources. Therefore, the Library of Alexandria would indeed need less shelf space than the Library of Congress. #### Yes.

**Question:** Can you transport a primate in a backpack?

**Originally incorrect reasoning from gpt-3.5-turbo:** Primates are a diverse group of animals that include monkeys, apes, and lemurs. Primates require specialized care and environment for transportation. Primates can be transported in specialized animal transport containers or crates. It is not advisable or ethical to transport a primate in a backpack. #### No.

**Corrected reasoning by COBB**: Primates are a diverse group of animals that includes monkeys, apes, and lemurs. Primates have various sizes and some can be quite large, like gorillas. Backpacks have limited space and are not designed to carry large items. It is possible to transport a small primate, like a baby monkey, in a backpack with proper care and precautions. #### Yes.

Figure 8: More qualitative examples on StrategyQA. Examples of the question, original reasoning from black-box LLM (gpt-3.5-turbo), and the adapted reasoning by COBB.

**Question:** On Tuesday, Peter wants to exercise for twice the amount of time he did on Monday and Sunday combined. On Sunday he exercised for 23 minutes. On Monday he exercised for 16 minutes. How many minutes does he have to exercise on Tuesday to reach his goal?

**Originally incorrect reasoning from gpt-3.5-turbo:** Peter exercised for a total of 23 + 16 = 39 minutes on Sunday and Monday. Therefore, on Tuesday Peter wants to exercise for  $39 \times 2 = 78$  minutes. Peter has to exercise for 78 - 23 - 16 = 39 minutes on Tuesday to reach his goal. The answer is 39 minutes.

**Corrected reasoning by CoBB**: On Monday, Peter exercised for 16 minutes. On Sunday, Peter exercised for 23 minutes. Combined, Peter exercised for 16 + 23 = 39 minutes on Monday and Sunday. To exercise twice as much as Monday and Sunday combined, Peter needs to exercise for 39 \* 2 = 78 minutes on Tuesday. Therefore, Peter needs to exercise for 78 minutes on Tuesday.

**Question:** 48 people are riding a bus. On the first stop, 8 passengers get off, and 5 times as many people as the number who got off from the bus get into the bus. On the second stop 21, passengers get off and 3 times fewer passengers get on. How many passengers are riding the bus after the second stop?

**Originally incorrect reasoning from gpt-3.5-turbo:** Initially, the bus has 48 passengers. After the first stop, 8 passengers get off, so there are 48 - 8 = 40 passengers remaining. 5 times the number of people getting off then get on the bus, which is 5 \* 8 = 40 passengers. So, after the first stop, there are 40 + 40 = 80 passengers on the bus. After the second stop, 21 passengers get off, leaving 80 - 21 = 59 passengers. 3 times fewer passengers than those who got off at the second stop get on, which is 3 \* 21 = 63 passengers. Thus, after the second stop, there are 59 + 63 = 122 passengers riding the bus. Therefore, there are 122 passengers on the bus after the second stop.

**Corrected reasoning by CoBB**: At the first stop, 8 passengers got off the bus, so now there are 48 - 8 = 40 passengers left on the bus. Then, 5 times as many people as the ones who got off got onto the bus, meaning 5\*8 = 40 more passengers entered the bus. This means there were originally 40 passengers on the bus + 40 new passengers = 80 passengers total on the bus after the first stop. At the second stop, 21 passengers got off the bus, so now there are 80 - 21 = 59 passengers left on the bus. Since 3 times fewer passengers got on compared to when 21 passengers got off, this means only 21/3 = 7 passengers got on at the second stop. Therefore, after the second stop, there are 59 passengers already on the bus + 7 new passengers = 66 passengers in total riding the bus.

**Question:** Nathan has a bouncy ball that bounces to 2/3rds of its starting height with each bounce. If he drops it from the third-floor balcony in the mall, where each story is 24 feet high, how high does the ball go on its second bounce?

**Originally incorrect reasoning from gpt-3.5-turbo:** Given that the bouncy ball bounces to 2/3 of its starting height with each bounce, on the second bounce it would reach 2/3 \* (2/3) = 4/9 of its original height. Let's calculate the height it will reach on the second bounce: - The third-floor balcony is 24 feet high. - On the first bounce, the ball reaches 2/3 \* 24 = 16 feet high. - On the second bounce, the ball reaches 4/9 \* 16 = 64/9 feet high. Therefore, the ball reaches a height of 64/9 feet on its second bounce.

Figure 9: More qualitative examples on GSM8K. Examples of the question, original reasoning from black-box LLM (gpt-3.5-turbo), and the adapted reasoning by CoBB.