Scaling Laws Across Model Architectures: A Comparative Analysis of Dense and MoE Models in Large Language Models

Siqi Wang^{1,2}, Zhengyu Chen^{1*}, Bei Li¹, Keqing He¹, Min Zhang², Jingang Wang^{1*}

¹Meituan Inc. ²The University of Hong Kong {chenzhengyu04,libei17,hekeqing,wangjingang02}@meituan.com {siqiwang,minzhang12}@hku.hk

Abstract

The scaling of large language models (LLMs) is a critical research area for the efficiency and effectiveness of model training and deployment. Our work investigates the transferability and discrepancies of scaling laws between Dense Models and Mixture of Experts (MoE) models. Through a combination of theoretical analysis and extensive experiments, including consistent loss scaling, optimal batch size and learning rate scaling, and resource allocation strategies scaling, our findings reveal that the power-law scaling framework also applies to MoE Models, indicating that the fundamental principles governing the scaling behavior of these models are preserved, even though the architecture differs. Additionally, MoE Models demonstrate superior generalization, resulting in lower testing losses with the same training compute budget compared to Dense Models. These findings indicate the scaling consistency and transfer generalization capabilities of MoE Models, providing new insights for optimizing MoE Model training and deployment strategies.

1 Introduction

The advent and scaling of large language models (LLMs), such as GPT (Brown et al., 2020; Achiam et al., 2023), Llama (Touvron et al., 2023a,b), Gemini (Team et al., 2023), Gopher (Rae et al., 2021), Chinchilla (Hoffmann et al., 2022), and Mistral (Jiang et al., 2023), have marked a transformative era in artificial intelligence and natural language processing. Characterized by their vast parameter counts and extensive training datasets, these models have significantly advanced capabilities across various domains, including machine translation (Brown et al., 2020; Hendy et al., 2023; Garcia and Firat, 2022), logical reasoning (Huang and Chang, 2022; Wei et al., 2022; Chen et al., 2021a, 2024a), and medical applications (Thirunavukarasu et al., 2023; Xiao et al., 2022). However, their increasing complexity and parameter scale posits an urgent need for innovative scaling strategies that optimize computational efficiency without compromising performance.

Historically, Dense Transformer Models have dominated due to their simplicity and scalability. And the scaling laws (Kaplan et al., 2020; Hoffmann et al., 2022) for dense models have been thoroughly investigated across different circumstances, such as over-training (Gadre et al., 2024) and datalimiting (Muennighoff et al., 2024; Chen et al., 2022). Despite their efficacy, the huge computational demands of these models necessitate exploration of alternative architectures like Mixture of Experts (MoE) (Yuksel et al., 2012; Shazeer et al., 2017; Du et al., 2022; Shen et al., 2024; Chen et al., 2021b; Xiao et al., 2021), which offer a promising reduction in computational load through sparse activations and dynamic expert routing.

This paper delves into the analysis of scaling laws for Dense and MoE Models within the context of LLMs. We extend foundational research on hyperparameters, such as compute budget, batch size, and learning rate (Kaplan et al., 2020; Hoffmann et al., 2022; McCandlish et al., 2018; Li et al., 2024; Chen et al., 2024b), to explore their transferability and applicability across these architectures. Our experiments involve models up to 7 billion parameters and datasets exceeding 100 billion tokens, aiming to uncover universal scaling behaviors potentially applicable to both model types.

Our results verify the hypothesis that certain scaling laws, particularly those related to loss and hyperparameters, may indeed be universal, bridging architectural gaps between Dense and MoE Models. This universality suggests a simplification in hyperparameter tuning across different scales and architectures, which could significantly streamline the training processes for various LLMs. Furthermore, we provide detailed analyses of the dif-

^{*}Corresponding authors.

ferential impacts of coefficient changes between MoE and Dense Models, offering both empirical and theoretical insights into the superior data efficiency of MoE Models. Concretely, MoE Models can achieve comparable performance with fewer training tokens than Dense Models, alleviating data constraints in LLM training. Our findings can be summarized as follows:

- Consistent Scaling Law Framework: Both MoE and Dense Models demonstrate a consistent and transferable scaling law framework, encompassing loss scaling as well as optimal batch size and learning rate scaling. This alignment implies that the established practices and insights for optimizing Dense Models can be readily applied to MoE Models, potentially streamlining the process of identifying optimal hyperparameters and reducing experimental complexity.
- Enhanced Data Efficiency in MoE Models: MoE Models demonstrate an approximate 16.37% improvement in data utilization over Dense Models under similar computational budgets. Theoretical and empirical analyses suggest that during training process, MoE Models, particularly when utilizing the Adam Optimizer, experience lower gradient noise scales. These results show that MoE Models could achieve stable training with smaller batch sizes and larger learning rates, potentially speeding up the training process and improving training convergence.

2 Related Work

Large Language Models Large language models (LLMs) such as GPT (Brown et al., 2020), Llama (Touvron et al., 2023a,b), Chinchilla (Hoffmann et al., 2022), Gopher (Rae et al., 2021), Mixtral 8x7B (Jiang et al., 2024), Switch Transformer (Fedus et al., 2022), GLaM (Du et al., 2022), and DeepSpeed-MoE (Rajbhandari et al., 2022) have advanced significantly, categorized into Dense Models and Mixture of Experts (MoE) Models. Dense Models activate all parameters per forward pass, while MoE Models activate only a subset, allowing for larger model scales without proportional increases in computational costs. Despite their complexity, MoE Models have shown potential for superior performance and efficiency. Scaling Laws for LLMs Due to the significant costs associated with training process, understanding the scaling laws of large language models (LLMs) is crucial. Studies (Bahri et al., 2021; Kaplan et al., 2020; Bi et al., 2024) have established a power-law relationship between model loss and factors like training tokens and compute budget. Recent work Yun et al. (2024) has explored these relationships further in MoE Models, indicating cost-effective scaling benefits but also highlighting challenges such as expert selection and load balancing. However, a systematic investigation into the scaling laws of MoE Models' hyperparameters and the transferability of scaling laws between Dense Models and MoE Models remains lacking, which is the focus of our work.

Hyperparameters Estimation As model sizes increase, precise optimal hyperparameter estimation becomes critical (Chen and Wang, 2021). Research McCandlish et al. (2018) has focused on optimizing batch size and learning rates to balance training speed and efficiency. Novel approaches (Yang et al., 2022, 2023) like Maximal Update Parametrization suggest that optimal hyperparameters for smaller models might scale to larger models effectively. Our study extends these insights to explore hyperparameter transferability between Dense and MoE Models, focusing on resource allocation, learning rate, batch size, and their transfer rules for Dense Models and MoE Models.

3 Preliminary

The scaling law of the training loss for Dense Models with respect to the number of training tokens and model size has been extensively studied (Kaplan et al., 2020; Hoffmann et al., 2022). Previous work (Hoffmann et al., 2022) has proposed the following scaling law (shown in Equation 1). To introduce the concept of model scale (the FLOPs divided by the number of training tokens) as N in our work, we denote model size (number of parameters) as P to avoid confusion.

$$\hat{L}(P,D) = \frac{A}{P^{\alpha}} + \frac{B}{D^{\beta}} + \sigma \tag{1}$$

s.t.
$$FLOPs(P, D) = C$$

where \hat{L} is the training loss, D is the number of training tokens, P is the model size (number of parameters), and σ represents the minimum achievable training loss due to the dataset's inherent noise.



Figure 1: The extrapolated scaling curves for 1.5B Mixture of Experts (MoE) models. This demonstrates that the proposed Loss Scaling Curve $\hat{L}(N, D, E) =$ $\frac{A}{N^{\alpha}E^{\gamma}} + \frac{B}{D^{\beta}} + \sigma(E < 100)$, fits well for MoE Models (eight experts). Specifically, D is the number of tokens and N is the model scale, which is compute budget (C)divided by D, instead of the model size. E is the number of experts and σ represents the random noise scale of dataset. A, B, γ , α and β are all coefficients.

C is the compute budget with an approximation $C = 6PD. \alpha, \beta, A \text{ and } B \text{ are all coefficients.}$

For MoE (Mixture of Experts) Models, previous research (Clark et al., 2022) has proposed a separable scaling law (Equation 2) for loss between model size and the number of experts.

$$\hat{L}(P, E) = a\log(P) + b\log(E) + d \qquad (2)$$

where P is the number of model parameters, E is the number of experts, and a, b, d are coefficients. This equals to Equation 3.

$$\hat{L}(P,E) = \frac{10^d}{P^a E^b} \tag{3}$$

When using Equation 2 to fit the training loss curve of MoE Models, Clark et al. (2022) claim that a decrease in performance has been observed given by expert scaling. Specifically, the value of b increases with model size (in Equation 2) when model size is large. This suggests that as the model size increases, the benefit from increasing the number of experts E will finally decrease. To enhance the fitting ability of the scaling laws for MoE Models, a quadratic interaction term is added, resulting in Equation 4.

$$\hat{L}(P, E) = a \log(P) + b \log(E)$$
$$+ c \log(P) \log(E) + d$$
$$= \frac{10^d}{P^a E^{b+clog(P)}} = \frac{10^d}{E^b P^{a+clog(E)}}$$
(4)

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4 **Estimating Resources Allocation Strategy Scaling**

4.1 Scaling Laws for Training Loss

Kaplan et al. (2020) and Hoffmann et al. (2022) originally proposed a scaling law for Dense Models, while Clark et al. (2022) extended this law to scenarios involving multiple experts (MoE Models). Upon closer examination of Equation 3 and Equation 4, we observed that when the number of experts (E) remains fixed, these equations can be simplified to the first term in Equation 1. Motivated by these insights, we introduce a unified scaling law for both Dense Models and MoE Models, represented by Equation 5. Specifically, since the decrease in performance is observed only when the number of experts (E) is large, we adopt the simplified one (Equation 2) for small E value (below 100). Besides, previous work (Bi et al., 2024) suggests replacing the model size P (number of parameters) with model scale N, which is the result of the FLOPs divided by the number of tokens in order to fit Equation 1 more accurately. Finally, we get Equation 5:

$$\hat{L}(N, D, E) = \frac{A}{N^{\alpha}E^{\gamma}} + \frac{B}{D^{\beta}} + \sigma \qquad (5)$$

s.t. FLOPs(N, D) = C

where L is the training loss, D is the number of training tokens, and N is the model scale, which is the non-embedding FLOPs (C) divided by D. E is the number of experts and we suggest E is smaller than 100. σ roughly estimates the natural noise of the dataset, representing the minimum achievable training loss. A, B, α , β and γ are coefficients.

In order to validate Equation 5, in our experiment, we fitted the training data for both 200M and 700M MoE Models (both with Eight Experts) to a curve. We then used this curve to predict the training loss scaling behavior of a 1.5B MoE model. The results, shown in Figure 1, demonstrate that the formula we proposed based on previous work is applicable to MoE Models when the number of experts is not large. It could be observed that the reduction in benefits from increasing E is minimal and the scaling equation stands within our experiment scope. Therefore, we adopted the simplified version. This formula suggests that the Equation 5 could equal to Equation 1, given a fixed number of experts, for MoE Models. This consistency could help us to compare the computing resource allo-



Figure 2: This diagram presents a heatmap of the distribution of training loss in relation to optimal batch size and training token quantities, with fitted curves representing different training loss. A vertical red line connects the minimum values of each curve. (a) training loss vs. optimal batch size when the MoE model size is 100M and the learning rate is 4e-3. (b) training loss vs. optimal batch size when the MoE model size is 700M and the learning rate is 1e-3.

cation strategy scaling for Dense Model and MoE Models (when E is given).

4.2 Estimating Optimal Resource Allocation Strategy Scaling

After fitting the training loss (L) as a function of the number of tokens (D), the model scale (N), and the number of experts (E), we proceed to derive the optimal computing resource allocation strategy for model scale and the number of training tokens given a fixed compute budget.

The objective can be defined as follows: given a fixed number of compute budget, how to estimate the optimal resource allocation strategy for model scale and the number of training tokens to minimize the training loss.

$$D_{\text{opt}}(C), N_{\text{opt}}(C) = \underset{D,N\text{s.t.}C=ND}{\arg\min} L(N, D, E)$$
(6)

We take the differentiation of Equation 5 with respect to C = DN, we derive the optimal values of D and N for a given compute budget:

$$\hat{D}_{opt}(C) = k_D \cdot C^{\alpha_D} \tag{7}$$

where $k_D = \left(\frac{\alpha A}{\beta B E^{\gamma}}\right)^{-\frac{1}{\alpha+\beta}}$ and $\alpha_D = \frac{\alpha}{\alpha+\beta}$

$$\hat{N}_{opt}(C) = k_N \cdot C^{\alpha_N} \tag{8}$$

where $k_N = \left(\frac{\alpha A}{\beta B E^{\gamma}}\right)^{\frac{1}{\alpha+\beta}}$ and $\alpha_N = \frac{\beta}{\alpha+\beta}$. Equation 7 and 8 indicate how the model scale (compute budget per token N) and the optimal number of training tokens D scale with the overall compute budget C. The most crucial part of this process is to

find the scaling exponent of the model scale and tokens number with reference to the compute budget. Then we use the empirical fitting results obtained previously to calculate these two exponents.

Table 1: Coefficients of optimal model and data scaling allocation for different model architectures.

Model Architecture	α_D	α_N
OpenAI (OpenWebText2)	0.27	0.73
Chinchilla (MassiveText)	0.51	0.49
DeepSeek (OpenWebText2)	0.422	0.578
Dense Model	0.493	0.507
MoE Model	0.410	0.590

From the data shown in Table 1, we observe that the exponent for the optimal model scale (N) in Mixture of Experts (MoE) models is larger, while the exponent for the optimal number of training tokens (D) is smaller, compared to their Dense Model counterparts. This suggests that MoE Models benefit more from increasing model size relative to the number of training tokens.

This finding helps explain why MoE Models can outperform larger Dense Models. The higher utilization of training data by MoE Models allows them to leverage their diverse sub-networks more effectively, capturing a broader range of features and patterns. Moreover, this suggests that it is more advantageous to allocate a larger computing budget to MoE Models compared to Dense Models.



Figure 3: We plot the optimal batch size values together with the corresponding training loss values across different model sizes for both Dense Models and MoE Models. In log scale diagrams, (a) demonstrates the log-log relationship of training loss vs. optimal batch size for Dense Models. (b) demonstrates the log-log relationship of training loss vs. optimal batch size for MoE Models. This indicates that the power-law relationship remains consistent not only across model sizes but also across model architectures. The total overlap of the comparative performance interval is about 65.8%.

5 Optimal Hyperparameters Scaling

5.1 Estimating Optimal Batch Size

Batch size is a crucial hyperparameter for the training process. Previous works (McCandlish et al., 2018; Kaplan et al., 2020) have investigated the scaling law between optimal batch size and loss for Dense Models. In this experiment, we conduct the analysis of both models.

To start, according to previous work (McCandlish et al., 2018), let $\Delta L_{opt}(B)$ denote the optimal improvement in the loss function when using a batch size *B* with the optimal step size $\epsilon_{opt}(B)$. It takes into account the noise introduced by the gradient estimation process. Then $\Delta L_{opt}(B)$ could be shown in Equation 9.

$$\Delta L_{\rm opt}(B) = \Delta L_{\rm max} \left(1 + \frac{B_{\rm noise}}{B}\right)^{-1} \quad (9)$$

where ΔL_{max} is the maximum possible improvement in the loss function when the true gradient is used without noise. Here, the noise scale B_{noise} measures the scale of the noise in the gradient estimates relative to the true gradient. It helps quantify how much the noise affects the gradient estimation process. The noise scale could be defined as:

$$B_{\text{noise}} = \frac{\text{tr}(H\Sigma)}{G^T H G} \tag{10}$$

where G denotes the true gradient and H the true Hessian at parameter values θ . Σ represents covariance matrix, and $tr(\cdot)$ represents the trace. A higher noise scale (B_{noise}) implies a larger optimal batch size for reducing the variance in the gradient estimate.

For training efficiency, the relationship between training steps S and training examples E derived from the SGD optimizer could be expressed as:

$$(\frac{S}{S_{\min}} - 1)(\frac{E}{E_{\min}} - 1) = 1$$
 (11)

where $S_{\rm min}$ and $E_{\rm min}$ are the minimum training steps and training examples needed to achieve a specific performance. Finally, from the empirical and theoretical verification (McCandlish et al., 2018; Kaplan et al., 2020; Hu et al., 2024), the optimal batch size at a specific training loss could be approximated using $B_{\rm opt} \approx B_{\rm noise}$, then we get Equation 12.

$$B_{\text{noise}} \approx B_{\text{opt}} = \frac{E_{\min}}{S_{\min}} \approx \frac{\lambda_B}{L^{\alpha_B}}$$
 (12)

where λ_B and α_B are both coefficients. B_{opt} is the optimal batch size given a noisy gradient and L is the loss value.

Equation 12 indicates that B_{opt} serves as the balance point between training speed and data efficiency. It represents the optimal trade-off between training speed and data efficiency. Furthermore, it indicates that as training progresses and the loss decreases, B_{opt} gradually becomes larger, indicating that larger batch size is required to maintain the balance between training speed and data efficiency as the model converges.



Figure 4: This diagram displays a heatmap showing the distribution of training loss in relation to optimal learning rates and training token quantities, with fitted curves representing different training loss values. A vertical red line connects the minimum points of each curve. (a) Training loss vs. optimal learning rate when the MoE model size is 700M and the batch size is 128 (the number of sequences of length 8192). (b) Training loss vs. optimal learning rate when the MoE model size is 1.5B, with the batch size being 128 (the number of sequences of length 8192).

Then we investigate the Equation 12 for both Dense Models and MoE Models (8 experts). Following the procedure in (Hu et al., 2024), we first generate contour lines representing configurations with an equivalent number of training tokens (Figure 2) for both Dense Models and MoE Models. From each contour line, we identify the points that exhibited the minimum training loss along with their corresponding batch size and learning rate. Subsequently, these optimal points are plotted in Figure 3(a) and Figure 3(b) to illustrate the relationship between training loss and batch size. We could observe that Dense Model has a larger optimal batch size exponent, which means for the same task and training loss value, the noise scale for Dense Model is larger than that of MoE Models.

Through the theoretical analysis (Equation 10 and 12), the noise scale B_{noise} is related to the variance of the gradient estimates and B_{opt} could be approximate to the noise scale. The observation that MoE Models have smaller optimal batch sizes for the same loss suggests that MoE Models can achieve the same optimization efficiency with less data, indicating a smaller noise scale. However, for Dense Models, larger optimal batch sizes for the same loss suggest that dense models need more data to achieve the same optimization efficiency, indicating a larger noise scale. This means that the gradient estimates are noisier in dense models for a given batch size.

5.2 Estimating Optimal Learning Rate

In previous work (McCandlish et al., 2018), the scaling relationship between optimal learning rate

and optimal batch size when using SGD optimizer are illustrated as Equation 13.

$$\epsilon_{\text{opt}}(B) = \epsilon_{\max} \left(1 + \frac{B_{\text{noise}}}{B} \right)^{-1}$$
 (13)

where $\epsilon_{opt}(B)$ represents the optimal step size that minimizes the expected loss from the noisy gradient. And ϵ_{max} represents the optimal step size that minimizes the loss function when using the noiseless true gradient G to update the parameters. It is defined as $\epsilon_{max} = \frac{|G|^2}{G^T H G}$. This relationship (Equation 13) shows that when B is relatively small, this Equation could be reduced as a nearly linear scaling (Granziol et al., 2022; Goyal et al., 2017). When the batch size is fixed, with optimal learning rate decreases with the increasing of noise scale.

$$\epsilon_{\rm opt}(B) = \epsilon_{\rm max} \left(\frac{B}{B_{\rm noise}}\right)$$

Recent research by Li et al. (2024) and Granziol et al. (2022) reveals an interesting trend in the optimal learning rate for the Adam Optimizer concerning the optimal batch size B_{opt} or noise scale B_{noise} . It shows a non-monotonic behavior, indicating that as the noise scale B_{noise} increases, the optimal learning rate initially follows a nearly square root scaling pattern, dominated by the second term, before decreasing as the first term gains dominance. The transition point occurs when these two terms reach a balance, a critical juncture determined by the specific values of B_{noise} and B.

$$\epsilon_{\rm opt}(B) = \frac{2\epsilon_{\rm max}}{\sqrt{\frac{B_{\rm noise}}{B}} + \sqrt{\frac{B}{B_{\rm noise}}}}$$
(14)



Figure 5: We plot the optimal learning rate values together with the corresponding training loss values across different model sizes for both Dense Models and MoE Models. In log scale diagrams, (a) demonstrates the log-log relationship of training loss vs. optimal learning rate for Dense Models. (b) demonstrates the log-log relationship of training loss vs. optimal learning rate for MoE Models. This indicates that the power-law relationship remains consistent not only across model sizes but also across model architectures. The total overlap of the comparative performance interval is about 76.2%.

In order to verify this, in this experiment (shown in Figure 4), we plot contour lines that represent configurations with an equivalent number of training tokens. For each contour line, we identify the points that achieved the minimum training loss, recording their corresponding learning rate. Then Figure 5 illustrates the relationship between training loss and optimal learning rate in Equation 15.

$$\epsilon_{opt} \approx \frac{\lambda_{\epsilon}}{L^{\alpha_{\epsilon}}} \tag{15}$$

where λ_{ϵ} and α_{ϵ} are both coefficients. ϵ_{opt} is the optimal learning rate given a noisy gradient and L is the loss value.

From the observation, MoE Models are likely to have a larger optimal learning rate compared to Dense Models when assuming the same loss value. Firstly, MoE Models tend to have smaller noise scale for the same loss value compared to Dense Models. A smaller noise scale indicates that the gradient estimates are less noisy, allowing for more accurate updates with smaller batch sizes. This efficiency with smaller batches also translates into requiring larger learning rate for effective optimization. Secondly, the optimal learning rate is roughly inversely proportional to the noise scale when the first term dominates for Equation 14. In conclusion, MoE Models are generally more efficient with smaller batch sizes and larger learning rates. It is very likely the model architecture that makes MoE models more efficient and more able to handle complex data.



Figure 6: We explore the generalization ability for both Dense Models and MoE Models. We highlight the stable power-law relationship interval across different model sizes for testing loss and the compute budget. We observe that with a comparable compute budget, MoE Models could achieve better testing loss.

6 Generalization of Scaling Law

We predict that MoE Models could have better generalization ability compared to their Dense counterparts. Firstly, MoE Models consist of multiple expert sub-networks that specialize in different aspects of the data, which enables the model to capture a broader range of features and patterns and then leads to better generalization across various tasks and datasets. Additionally, the ensemble nature of MoE Models, where each expert contributes to the final prediction, reduces overfitting and improves robustness by combining predictions from multiple models. Finally, the gating mechanisms in MoE Models control the contribution of each expert to the final prediction, acting as a form of

Table 2: Details about the performance of our Dense and MoE Models involved in all experiments.

Model Size	TriviaQA	MATH	MMLU	CMMLU	MATH401
Dense-7B	54.67	5.26	28.73	25.03	49.13
Dense-1B	20.56	1.48	22.42	21.38	5.99
Dense-500M	17.86	1.18	14.65	20.02	4.99
Dense-100M	14.28	0.66	14.68	18.71	3.74
Dense-50M	13.47	0.96	11.17	22.84	1.75
MoE-3B	56.09	6.74	29.18	25.46	40.12
MoE-1.5B	26.25	4.24	25.26	19.81	7.02
MoE-700M	18.89	1.80	15.51	18.04	4.68
MoE-200M	14.36	0.96	14.40	14.60	2.77

regularization. This regularization helps prevent overfitting by focusing on the most relevant experts for each input, resulting in better generalization.

To investigate the generalization performance of the scaling law for both MoE Models (Eight Experts) and Dense Models, we explore and compare the relationship of training loss with testing loss and compute budget for both Dense Models and MoE Models (shown in Figure 6). We highlight the stable power-law relationship interval across different model sizes, illustrating the correlation between testing loss and compute budget. Notably, MoE Models consistently exhibit a smaller testing loss for a given compute budget, indicating their superior generalization performance.

We also explore the performance of the Dense and MoE Models on different testing sets in Table 2. It clearly shows that MoE Models could outperform a Dense Model with comparative model size. The consistent trend in performance across different datasets underscores the transferability and reliability of the scaling law observed in our study. This finding suggests that the performance improvements achieved by MoE Models are not limited to specific datasets or conditions but hold true across diverse testing sets, indicating robustness and generalizability in real-world applications.

7 Conclusion

In this paper, we investigate the transfer of traditional scaling laws from Dense Models to Mixture of Experts (MoE) Models. Our investigation confirms that the power-law relationship extends to MoE Models regarding the consistency and transferability of scaling strategies, including resource allocation strategy, optimal batch size / learning rate scaling, and so on. This observation indicates that the fundamental principles of training dynamics and the behavior of scaling rules are similar for both Models. This means existing knowledge and practices for optimizing Dense Models can be easily adapted to MoE Models, potentially reducing the experimental burden of finding the best hyperparameters. Besides, we find that MoE Models demonstrate approximately a 16.37% improvement in data utilization efficiency compared to Dense Models with a fixed compute budget. Thus we suggest prioritizing increasing model scale over other factors when training MoE Models, highlighting their greater data efficiency. Finally, we use both theoretical and empirical analysis to reveal that during the training process, MoE Models exhibit a lower gradient noise scale when using the Adam Optimizer. At the same training loss value, MoE Models can achieve stable training with smaller batch sizes and larger learning rates, potentially speeding up the training process and improving convergence. It shows that MoE Models can make more efficient use of training data and computational resources, extracting more information per training token, leading to faster training times and better utilization of available data. These results offer valuable insights for refining training and deployment strategies for MoE Models.

Limitation

The experiments were constrained by the available computational resources. Firstly, we have not yet explored scenarios with more than 100 experts in our experiments. It has been observed that as the model size increases, the marginal benefit from increasing the number of experts tends to decrease. Moreover, although we tested our model on several benchmark datasets, they don't cover all domains. To ensure robust evaluation, we plan to validate the model on more tasks and domains and investigate the scaling relationships between different metrics. Additionally, as the number of training tokens exceeded 100 billion, we observed signs of overtraining for both models, indicating diminishing returns in performance improvements with additional training data. These areas remain open for future research.

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A Experimental Settings

The architecture of all models (including Dense Models and MoE Models) are shown in table 3 and table 4, respectively. The number of layers, attention heads, hidden dimensions, and other relevant details are listed in the tables.

During training, we employed the AdamW optimizer with parameters $\beta_1 = 0.9$ and $\beta_2 = 0.95$ for all models. Following the Chinchilla law (Hoffmann et al., 2022), we established a maximum learning rate of 1.5×10^{-3} for smaller models and 2×10^{-4} for larger ones. A cosine scheduler with a 10x learning rate decay was implemented throughout the training process. We applied Gaussian smoothing with a 10-step window length to enhance the training curve.

Specifically, we identified the best performance values within our hyperparameter range. The range of hyperparameter settings, including batch size and learning rate, was carefully selected for each model size to ensure optimal performance within the designated FLOP budget. Our observations indicate that performance tends to converge to optimal values around the neighborhood of the best settings, as illustrated in Figure 7.



Figure 7: We observed that the optimal settings are a Learning Rate of 1.5e-3 and a Batch Size of 256. Additionally, in the neighborhood of these settings, the training loss values have an error within 2%, indicating that the model will converge to the optimal value around the best hyperparameter settings.

The dataset we used for the training of Dense Models and MoE Models is The Pile (Gao et al., 2020), which is a 825 GiB English text corpus consisting of 22 high-quality subsets.

Table 3: Details about the architecture of our Dense Models involved in all experiments.

Model Size	Params.	Hid. Size	Layers	Head Size	FFN
50M	47.71M	512	14	4	1536
100M	113.25M	768	16	6	2048
500M	487.59M	1280	24	10	3584
1B	936.31M	1664	28	13	4480
7B	7.03B	4096	32	32	11008

Table 4: Details about the architecture of our MoE Models involved in all experiments, all of them are top-3.

Model Size	Act. Params	Hid. Size	Layers	Head Size	Int. Size
200M	183M	640	12	8	1792
700M	692M	1280	16	16	3456
1.5B	1.45B	1600	24	20	4352
3B	3.0B	2304	28	32	6272

B Discussion of related works

Large language models (LLMs) have obtained significant attention and undergone substantial development in recent years. They can be categorized into two main classes based on their parameter utilization during the forward pass: Dense Models and MoE (Model of Experts) Models. Dense Models, such as GPT-2 (Radford et al., 2019), GPT-3 (Brown et al., 2020), Llama (Touvron et al., 2023a,b), Chinchilla (Hoffmann et al., 2022), and Gopher (Rae et al., 2021), maintain a total parameter count equal to the active parameter count per forward pass. On the other hand, MoE Models (Shazeer et al., 2017) activate only a subset of total parameters during training, as seen in models such as Mixtral 8x7B (Jiang et al., 2024), Switch Transformer (Fedus et al., 2022), GShard (Lepikhin et al., 2020), GLaM (Du et al., 2022), and DeepSpeed-MoE (Rajbhandari et al., 2022). Dense Models are known for their simplicity in implementation and training. However, MoE Models can scale to significantly larger total parameter counts without a proportional increase in computational cost. Despite challenges like load balancing and expert selection faced by MoE Models, prior research indicates that they offer superior performance due to increased model capacity and enhanced data efficiency.

Due to the significant costs associated with training process, understanding the scaling laws of large language models (LLMs) is crucial. Some previous studies (Kaplan et al., 2020; Bahri et al., 2021) have proposed a power-law relationship between loss and various factors like the number of non-embedding parameters, training tokens, and compute budget across different magnitudes. Notably, Kaplan et al. (2020) found that increasing the model size by 8 times only requires a roughly 5x increase in data to avoid penalties. In contrast to earlier findings, Hoffmann et al. (2022) implements optimized training configurations, which include the use of training tokens and learning rate schedules, and recommends scaling training tokens in proportion to model size. Additionally, research by Bi et al. (2024) explores the scaling laws of batch size and learning rate in relation to model scale (non-embedding FLOPs per token), offering a more precise estimation. They propose an allocation strategy for scaling up models and data based on dataset quality. While some studies (Li et al., 2024; McCandlish et al., 2018) link the optimal batch size to gradient noise scale and optimizer type, recent attention has shifted towards Mixture of Expert (MoE) models due to their potential cost savings. Fedus et al. (2022) investigates the scaling properties of MoE Models and suggest that having more parameters (experts) with a fixed computational budget accelerates training and surpasses dense Transformer baselines. On the other hand,

Yun et al. (2024) incorporate the hyperparameter E (number of experts) into existing scaling laws, revealing diminishing returns with increasing expert numbers. However, a systematic investigation into the scaling laws of MoE Models' hyperparameters and the transferability of scaling laws between Dense Models and MoE Models remains lacking, which will be the focus of our work.

With the increasing size of models in deep learning, hyperparameter estimation has gained significant attention due to the huge costs associated with hyperparameter tuning. In training large language models, several key hyperparameters require careful consideration and selection. Previous research has offered valuable insights that inspire our approach. For instance, McCandlish et al. (2018) highlights the trade-off between training speed and efficiency, focusing on the critical batch size, which can be measured by the gradient noise scale. Other studies (Goyal et al., 2017; Granziol et al., 2022; Li et al., 2024) propose a linear scaling rule for adjusting learning rates as a function of minibatch size for SGD optimizers, while a square root scaling rule is suggested for adaptive optimizers. Some works focus on optimal hyperparameter transfer. For instance, unlike traditional Bayesian Optimization (BO)-based methods (Horváth et al., 2021; Perrone et al., 2018), recent studies (Yang et al., 2022, 2023) emphasize transferring hyperparameters across model scales. They introduce a novel zero-shot strategy known as Maximal Update Parametrization (μ P), demonstrating that optimal hyperparameter choices for smaller models remain effective for larger ones. However, there is a lack of research on hyperparameter transfer across MoE Models and Dense Models. Our work specifically emphasizes critical hyperparameters transfer such as resource allocation, learning rate, and batch size, and their transfer rules for Dense Models and MoE Models.

C Details of Differentiation

Given the loss function:

$$\hat{L}(N, D, E) = \frac{A}{N^{\alpha} E^{\gamma}} + \frac{B}{D^{\beta}} + \sigma \qquad (16)$$

and the constraint:

$$C = ND \tag{17}$$

where N, D, C and E are the model scale (nonembedding FLOPs per token) and the number of training tokens, compute budget and the number of experts respectively. A, B, α , β , γ are all coefficients. σ is the noise of the dataset, which is the minimum loss value for model.

We can substitute $D = \frac{C}{N}$ into the loss function. Next, we differentiate \hat{L} with respect to N:

$$\hat{L}(N, \frac{C}{N}, E) = \frac{A}{N^{\alpha} E^{\gamma}} + \frac{BN^{\beta}}{C^{\beta}} + \sigma \qquad (18)$$

Next, we differentiate \hat{L} with respect to N:

$$\frac{d\hat{L}}{dN} = \frac{d}{dN} \left(\frac{A}{N^{\alpha} E^{\gamma}} + \frac{BN^{\beta}}{C^{\beta}} + \sigma \right)$$
(19)

The N_{opt} and D_{opt} are the critical point when L achieves the minimum loss value. Set the derivative to zero to find the critical points:

$$-\frac{\alpha A}{N^{\alpha+1}E^{\gamma}} + \frac{\beta B N^{\beta-1}}{C^{\beta}} = 0$$
 (20)

Therefore,

$$N_{\text{opt}}(C) = \left(\frac{\alpha A}{\beta B E^{\gamma}}\right)^{\frac{1}{\alpha+\beta}} C^{\frac{\beta}{\alpha+\beta}} \qquad (21)$$

Using the constraint C = ND, we find D_{opt} :

$$D_{\rm opt}(C) = \frac{C}{N_{\rm opt}} \tag{22}$$

$$D_{\text{opt}}(C) = \frac{C}{\left(\frac{\alpha A}{\beta B E^{\gamma}}\right)^{\frac{1}{\alpha+\beta}} C^{\frac{\beta}{\alpha+\beta}}}$$
(23)

Therefore,

$$D_{\text{opt}}(C) = \left(\frac{\alpha A}{\beta B E^{\gamma}}\right)^{-\frac{1}{\alpha+\beta}} C^{\frac{\alpha}{\alpha+\beta}}$$
(24)